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R. P. S.

Investigation of some Trouble

in the

Generating System

of the

Commonwealth Edison Co.

Chicago

1919

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Schenectady, N. Y., December 19, 1919.

Mr. S. Insull, Pres.
Commonwealth Edison Company,
Chicago, Ill.

My dear Mr. Insull:

Enclosed I send you report of investigation of some operating troubles in the generating system of the Commonwealth Edison Company, during 1919, with some recommendations. I am sending copies of the report to Mr. L. Ferguson and to Mr. R. F. Schuchardt. I regret that in some respects the report is not as final and conclusive as I like to see it, but during the years of successful operation since the installation of the protective reactances, your system has grown and changed so much, and while I have received very complete and extremely satisfactory information and data from your engineers, it necessarily is not possible for me to be as fully familiar with the system, as I was once, but I hope that I shall now be able to keep in closer touch with it.

Some of my recommendations therefore are more general, and require further study by the operating engineers, and I shall be glad to co-operate therein, and expect to be in Chicago again in January. More particularly this applies to:—

1.) The installation of power limiting reactors between the Northwest Station and Fisk Street, which appears to me extremely desirable to eliminate the excessive interference between these stations in case of trouble in one of them. As, however, the tie cables between these stations are also used as feeder cables for intermediate substations, a study by your engineers, in which I shall be glad to co-operate, is necessary to devise an arrangement of installation of reactors, which would not interfere with the economic use of tie cables of substation feeder cables.

2.) The substations were originally operated by separate feeders, but the need of using the feeder cables and apparatus in the most economical manner has led to tying substations together on the same feeder, which necessarily increases the interference between substations and between generating stations in case of trouble, and further changes are contemplated and desirable. It therefore appears advisable to make a thorough study of substation operation with the view of combining the most economical use of cables and apparatus with the minimum possibility of interference between stations in case of local trouble. Such study can be made only by the operating engineers who are fully familiar with all the conditions, but I shall be glad to co-operate in it.

One suggestion which I like to bring to your consideration is that of building a typical model of your system, generating stations, cables and substations, perhaps in 1/20,000 size. We have done that for a number of transmission systems and while your system is far larger and more complex, I believe it can be done and such model, operated by direct current, may be contained in a large room, and while not all, many operating questions could be studied on it by the engineers, and the effect of various disturbances and troubles experimentally investigated without endangering your big system.

With best regards,

Yours very truly,

(Signed) CHARLES P. STEINMETZ.

CPS:R

INVESTIGATION OF SOME TROUBLE IN THE GENERATING SYSTEM OF THE COMMONWEALTH EDISON CO. OF CHICAGO, DURING 1919

By Charles P. Steinmetz, A.M., Ph.D.

I—RECOMMENDATIONS

From the investigation, the following recommendations appear to me justified:

1.) To *reduce the liability of trouble,* by carefully going over all the controlling devices, such as relays, current transformers, circuit breaker-operating mechanisms, etc., especially those at or near the generating stations to ascertain whether they are in perfect condition and whether they are of the most reliable and safest type now available, and where necessary replace them or change them to the safest and most reliable now available for the existing conditions of operation. It must be expected, that during the time which many of the controlling devices have been in operation in the system, advances have been made in type and design of circuit controlling devices. The conditions of operation have become more severe, due to the increase of the size of the system and especially due to the increasing interconnection of substations by tie cables. While such interconnection materially increases the economy in the use of the cables, it also increases the severity and extent of local troubles such as short circuits. Furthermore, many of the controlling devices are not new any more.

While it is economically not feasible to replace or remodel the controlling devices every few years with every advance of the art, it probably is economically feasible to do so with regard to the controlling devices located in the generating stations proper.

2.) To study the possibility of *intercepting many of the troubles in their beginning,* before they have fully developed into a short circuit. Cable breakdowns apparently are not always instantaneous, but often

develop gradually within a time from a few seconds to many days. A sufficiently sensitive differential relay thus may discover a beginning cable fault, and cut off the cable, before the fault has developed into a ground or short. In a split conductor cable, the two parts of each conductor are so closely identical, that a very sensitive differential relay can be placed between them, and as a fault naturally would develop in one of the cable halves first, the relay would act at the very beginning of the fault. The possibilities and limitations, and in general the economic feasibility of the split conductor cable, should thus be investigated. Similar results are given by grouping in pairs of identical cables with differential relays between them. This latter arrangement perhaps is somewhat less sensitive and reliable, since with two separate cables, no matter how identical they may be, a transient may occur in the one and not—or a different transient—in the other, and the sensitivity of the differential relay thus probably has to be lowered not to be affected by transients. On the other hand, the latter arrangement would not require such extensive replacement of cables. Systems involving the use of sheath transformers or other schemes for tripping out on small ground currents, and still other arrangements for accomplishing the result of operating on an incipient fault, should be investigated.

It appears that some cable failures are preceded by a gradual decrease of the insulation resistance, especially while hot, extending over many days. Such failures might be intercepted by a systematic testing of the cables with high voltage direct current, essentially a high voltage resistance measurement, and the possibility of such should be investigated. The time for such tests should be chosen immediately after the peak loads of the day, when the cables are at their maximum temperatures. I understand that simple devices for getting high voltage direct current for testing purposes have been developed.

3.) To *cut off the troubles from the generating stations* by the installation of *feeder reactances.*

By far the largest majority of troubles leading to short circuit occur in the feeder cables and beyond them, in the substations, but very few only in the generators, and extremely few on the busbars. The generators have power limiting reactors, but no power limiting reactors are used in the feeders, and as the result, any short circuit in a feeder cable, near the generating station, is practically a short circuit on the busbars, that is, pulls the voltage of the station section down to nothing, drops out the synchronous apparatus and thus gives serious and wide-

spread trouble. Such short circuits in the feeder cable near the generating stations, however, may be expected to be more frequent than short circuits in the generating station itself, and the installation of reliable feeder reactors thus would eliminate the majority of short circuits from materially affecting the generating stations, that is, from becoming serious.

I would recommend that 0.9 ohm, or at least 0.7 ohm feeder reactors (5.2% to 4% for a 300 ampere line) be installed, and the circuit breakers be set to cut off as quickly as possible, in case of a short circuit in the feeder. I believe such a feeder reactance would in no way adversely affect the operation of the substations, but it would limit the short circuit to about 100,000 KVA. If then the circuit breakers can be made to open this short in less than a second, the station voltage will be only a little affected during the short, due to the great sluggishness of the turbo-alternator fields, and immediately come back to practically normal, so that it may be expected that no synchronous apparatus will be dropped out, that is, the trouble limited to the short circuited feeder cable and its substations. If, however, the short circuit holds on for several seconds, an appreciable voltage drop must be expected in the generating stations, and at least some of the synchronous apparatus supplied from this generating station would be dropped out.

The advantage of feeder reactors thus not merely consists in limiting the short circuit current and thereby the voltage drop and in general the shock on the system, but, by permitting to set the circuit breakers for a materially shorter time limit, it also greatly reduces the duration of such short circuit and thereby correspondingly reduces the liability of dropping synchronous apparatus and spreading the trouble beyond the feeder directly involved.

4.) Install a *power limiting busbar reactance* between the two sections of Fisk Street Station, so as to tie the three station sections: Fisk Street A, Quarry Street and Fisk B, together into a ring. This should increase the synchronizing power between these stations. It should also guard against the system being cut into two parts out of synchronism with each other, in case that a short circuit at the busbars of an intermediary section (Quarry Street or Fisk Street B), drops the voltage of this section to zero and thereby destroys its synchronizing power.

The same size of reactance as now used, of about 1.75 ohms, would be recommended.

5.) I should recommend strongly to endeavor to change the present connection between the Northwest Station and the rest of the system, which now consists of six cables to Fisk Street B; and to connect the Northwest Station by cables and power-limiting reactances to Fisk Street A, as well as Fisk Street B, thus making a second ring, between Fisk Street B, Northwest Station and Fisk Street A, that is, have the entire system of four station-sections tied together into a double ring, by five power limiting reactors.

At present, due to the absence of power limiting reactances and the low resistance (.3 ohms) of the tie cables between the Northwest Station and Fisk Street B, these two stations are practically on the same busbars. This imposes too severe a duty on the controlling devices such as circuit breakers, of these two stations, and a trouble in one of these two stations is equally severe in the other, that is, a short circuit on the busbars of either station puts both stations out of service and thereby involves too large a part of the entire system, as borne out by the experience of September 18th.

As the present six tie cables between Northwest Station and Fisk Street B, are also used as feeder cables to intermediary substations, in the proposed ring connection and installation of reactances, careful study must be given to make the arrangement such as not to interfere with the economical use of these tie cables as feeder cables to the intermediary substations. Possibly three (or even two) of these cables may be used between Northwest Station and Fisk B; three (or even two) between Northwest Station and Fisk A; these three (or two) cables brought together at the station end to a short auxiliary bus and a power limiting reactance installed between this auxiliary bus and the main bus of the station. This would divide the power limiting reactance into halves, one at each end, thus giving a total of four new reactances, two at the Northwest Station, one at Fisk A, and one at Fisk B, for the interconnection with the Northwest Station. Each of the reactors then would be about .875 ohms (half the size of the present power limiting busbar reactors). Such an arrangement may require a slight increase of excitation of the synchronous converters in the substations connected to these tie cables, to keep their voltage by giving the current a slight lead. Another possibility, which might be more convenient, would be to install normal feeder reactors at each end of each of the tie cables, and install the rest of the required reactance in the substation, arranged so that the substation or the individual converters in the substations can tap the feeder cable at either side of this reactance, depending on from which station it intends to take the power. That is,

two busbars may be used in the substation, connected together by the power limiting reactance, the one connected to the one, the other to the other generating station, and the converters arranged so that they can be thrown on either of the two busbars. Possibly a still better arrangement may be devised.

Until reactors are installed between the Northwest Station and the rest of the system, I would recommend, whenever there is any serious trouble in the Northwest Station, or in Fisk Street B, which connects with it, to immediately open all the tie lines between these two stations, and synchronizing them together again after the trouble is perfectly cleared.

6.) Install in each station section, as permanent busbar instruments, as many suitable *synchronoscopes* as there are other station sections (three at present), for the purpose of continually indicating the phase difference and the frequency difference of the station section from all other station sections. If by some trouble a station section has broken out of synchronism with the rest of the system, it appears practically impossible without the assistance of a synchronoscope, to control the steam supply in this station section so as to have it promptly drop back into synchronism. With a synchronoscope, however, indicating the speed difference of the station from the next adjoining station, with which it is out of synchronism (though still tied with it by the reactance), merely the ordinary action of synchronizing will bring the station quickly back into synchronism.

Such synchronoscope would also indicate the phase difference between adjoining stations due to the power flow over the busbar reactances, and thereby permit most economical load control. Such synchronoscope would be very valuable in case of trouble, in showing what happens: whether the stations are out of synchronism, or hunting against each other, or steady but with excessive power flow over the reactors, etc. It is possible that a more sensitive type of synchronoscope will have to be designed, than the present one.

7.) For the present, until 4, 5 and 6, have been carried out I would recommend, in case the voltage of a station section disappears, as result of a short circuit at or near the station, and if the voltage does not promptly come back after the clearing of the short circuit, to open the power limiting reactor or reactors which connect this station section with the rest of the system, and thereby isolate it. Then, as soon as the voltage has recovered, the isolated station should again be synchronized in with the rest of the system. If, after thus isolating the

generating station in which the trouble has occurred, the voltage does not promptly recover, it means that the individual generators of this station have broken out of synchronism, and the quickest way of restoring service probably is, to disconnect the generators from each other, and synchronize them again.

8.) A special study should be made of the method of operation of the substations, that is, their connection to the generating system and to each other, to get the most satisfactory compromise beween reliability and economy; that is, to use the feeder copper most economically, and at the same time in case of trouble reduce to a minimum the interference of the substations with each other, and of the generating station sections with each other through the substations.

The question then requires consideration, whether and how far tie feeders between substations are to be used, and how they should be controlled and protected. Whether every important substation should receive power from two generating station sections, and whether and what protection in this case can be afforded against interference between the two substation sections in case of trouble in any generating station section. Or whether each substation should be fed from one generating station section only, but adjacent substations connected to different generating station sections, so that in case of a substation shutting down by trouble in the generating section feeding it, the adjacent substation can maintain service, etc.

Also, the question of the control of the converters in the substations should be investigated, whether the A.C. circuit breakers might be set somewhat higher; whether the D.C. reverse current relay may not be given a time limit and its setting increased; whether a D.C. power limiting resistance might be considered, etc.

Discussion of Recommendations

While recommendations 1) to 3) should greatly reduce the frequency of troubles or keep them out of the generating system by isolating or localizing them by the feeder reactors, it obviously is not possible to absolutely guard against the occasional troubles in the generating system, such as short circuits. But as soon as the trouble is cleared as by the opening of the circuit breakers, in a second or a few seconds, the system should immediately return to normal, and to begin to pick up again the load which the short circuit dropped. The most serious feature of the troubles of September 18th, May 19th, and October 22nd, in my opinion, was that with the clearing of the short circuit, the sys-

tem did not promptly come back to normal voltage, but in a large part of the system (Fisk Street B and Northwest) the voltage remained practically zero for about a quarter of an hour after the trouble had been cleared. It appears, as the result of the momentary short circuit the stations had broken out of synchronism with each other and were not able to pull back into synchronism, but kept drifting past each other indefinitely, short circuiting each other and thus keeping the voltage down to practically zero.

In these very large power systems, it is essential for the safety of operation to limit the possible local concentration of power, by dividing the system by power limiting reactors. To fulfill their purposes, these reactors must be fairly large, and the value of 1.75 ohms used in the power limiting busbar reactors of the Commonwealth Edison Company of Chicago, is by no means too high. Necessarily, however, these power limiting reactors also limit the synchronizing power between the station sections. Thus if in a station section as Fisk Street A, which is connected by one power limiting reactor to the rest of the system, full load of 60,000 KW is suddenly thrown off—as by a short circuit at the busbars dropping out the synchronous machines in the substations—while full steam supply is still on, the synchronizing power coming over the power limiting reactor is insufficient to hold the station in step, and the station breaks synchronism and speeds up. Whether synchronous operation is preserved or synchronism broken, depends on the relative speed, with which the synchronous machines in the substations drop out, the turbine governors shut off steam and the alternators speed up. The synchronous machines in the substations, carrying load on the direct current side and feeding back on the alternating side, probably would drop out very quickly, while the turbine governors must take an appreciable time to reduce the steam supply, and the alternators speed up rapidly; at full voltage and full steam supply, a little over a second after the load has dropped off, the stations would have speeded up so as to have broken synchronism with the rest of the system, in spite of the maximum synchronizing power exerted over the power limiting reactor. In reality obviously the load cannot drop off instantly but would hold on an appreciable time, and the governors would immediately begin to cut off steam; but on the other hand, the station voltage has dropped under short circuit, and is below normal, and with it the synchronizing power (which goes with the square of the voltage). Tying the station section by power limiting reactors to two other station sections (as by installing a reactor between Fisk Street A and B, thus completing ring connection) would give

ample synchronizing power, at full voltage, to keep the station section in synchronism with the rest of the system, even at no load but full steam supply, so that it could break out of synchronism only if the short circuit lasts sufficiently long to demagnetize the alternator fields and thereby drop the voltage.

Therefore it is recommended to tie all the stations by power limiting reactors into ring connection.

If a short circuit occurs at or near the busbars of a station section, it necessarily drops the busbar voltage to zero. It takes, however, a number of seconds for the short circuit current to demagnetize the alternator fields, and if therefore the short circuit is opened quickly, the alternator field magnetism is still there, at least partly, and the station voltage thus comes back instantly, at least partly. If then the station section has sufficient synchronizing power against the adjacent section, it is probable that it would remain in synchronism, no further trouble would occur, and it would probably catch again many of the synchronous machines receiving power from it.

If, however, the short circuit lasts long enough to materially demagnetize the alternator fields, then at the clearing of the short, the voltage does not immediately come back, as the field magnetism would first have to build up. Without voltage there obviously can be no synchronizing power, and the station section thus probably drifts out of synchronism with the rest of the system. With the load being released by the dropping out of the synchronous machines in the substations, before the governors can cut off steam, the turbo-alternators will probably have speeded up and operated their emergency steam cut off, as with full steam and no load this takes only about 1½ to 3 seconds. The machines then slow down until again put on governor control. As there are necessarily very great differences between the individual machines in the speed at which they trip the excess speed cut off, in the time required to reach this speed, and in the rate of slowing down, it is obvious that when governor control is restored, the individual machines and the station section as a whole probably are far from synchronism with the rest of the system. It therefore is hardly to be expected that they would promptly drop into synchronism but rather would continue indefinitely to drift out of synchronism with the rest of the system.

Two alternators or stations, thrown together out of synchronism, that is, differing in frequency from each other, will promptly, that is, practically instantly, pull each other into step, that is, the slow machine

speeds up and the fast machine slows down, if their frequency difference was low enough. This, however, would, with turbo-alternators, require a frequency difference not much exceeding one percent, and it is not probable that the unloaded station, idling on the governors, would be so close in frequency to the loaded station, especially as the frequency difference in normal steam governing of these turbo-alternators is 4% between no load and full load. If the frequency difference is greater, the two stations continue drifting past each other out of synchronism, but steadily a power transfer between them tends to pull them nearer together, so that, even with an initial frequency difference of 5%, they should pull each other into synchronism, in about a minute, more or less. However, this synchronizing power is so small, only a fraction of 1% of the rated load, and much less than the excitation or friction and windage losses in the machines, so that the inevitable variations in the steam supply when governing at no load, probably are much larger and overshadow this synchronizing power. That is, it has only a theoretical interest, and practically the stations would indefinitely drift past each other out of synchronism; with the fluctuations of steam supply, etc., sometimes coming nearer together, sometimes drifting farther apart, etc., until at some time they happen to drift close enough together—within 1%—so as to pull each other in step.

The characteristic of this drift out of synchronism is that the fluctuations of current, etc., are constant, and not gradually decreasing, as in hunting oscillations, and the frequency or period of fluctuation is irregular. This seems to agree with the observations.

It appears then: if a station section has dropped out of synchronism by a short circuit or other trouble, as indicated by its voltage not coming back promptly at the clearing of the short, then it is not probable that this station section will pull itself into synchronism within reasonable time, but regular synchronizing appears necessary.

Either the reactor or reactors connecting this station section should be opened, thus isolating this station and allowing it to again build up its voltage, and then it should be synchronized in again with the rest of the system.

Or, without opening the reactors, the station may be synchronized into the system by controlling the steam supply in correspondence with the indications of synchronoscopes connected between this station and the adjacent stations.

Similar drifting past each other, out of synchronism, may also occur, as the result of a disturbance such as a short circuit, between the turbo-alternators of one station, especially if they are machines of different types, as in the Northwest Station. Or a single machine may break out of synchronism, while the other machines in the same station stay in step—especially if the off machine is of different type, as No. 11 in Fisk Street. As there are no power limiting reactors used between the machines of the same station, the quickest way of restoring synchronism would probably be, in the first case, to open the circuit breakers of the individual machines and then synchronize them again; in the latter case, to open the circuit breakers of the off machine, and synchronize it again.

II—RECORD

Four troubles were studied, occurring respectively on—

September 18th,	1919,	3:47 P. M.
September 18th,	1919,	5:27 P. M.
October 22nd,	1919,	12:20 P. M.
May 19th,	1919,	7:25 A. M.

The generating system is divided into four sections, connected in tandem, with the A section of Fisk Street, and the Northwest Station as the two ends of the chain, and with power limiting reactors stated to be 1.75 ohms each, between Fisk A and Quarry Street, and between Quarry Street and Fisk B, and six tie cables of negligible reactance and about .3 ohms joint resistance between Fisk B and the Northwest Station.

1.) Sept. 18th, 1919—3:47 P. M.

a) A short circuit close to the busbars of B section of Fisk Street held on for several seconds, before it was opened.

As there are no power limiting reactors between Fisk B and Northwest Station, and the six tie cables between these stations are of very low resistance, the Northwest Station was just as seriously affected as Fisk B, and indeed acted like a part of Fisk B.

b) All the synchronous machines on Fisk Street B, and on Northwest Station dropped out, and many synchronous machines on Fisk A and Quarry Street: 44 synchronous machines on Fisk B and 18 on Northwest are recorded as shut down. Of 26 machines on Fisk A, 12 dropped out and 14 stayed in; of 8 machines on Quarry Street, 4 dropped out and 4 stayed in.

c) Due to the voltage dropping to zero under the short circuit, the turbo-alternators in Fisk B and Northwest Station dropped out of synchronism with each other, and out of synchronism with Quarry Street and Fisk A; but Quarry Street and Fisk A remained in synchronism with each other.

d) Due to the load being taken off by the dropping out of synchronous machines, the turbo-alternators in Fisk B and in the Northwest Station speeded up until the emergency governors tripped the steam valves. The turbo-alternators were put back on the governors immediately.

e) The turbo-alternators in Fisk B and in Northwest Station did not pull into step with each other, but remained out of synchronism; the voltage at the busbars of these two stations remained practically zero, and an excessive current fed into Fisk B from Quarry Street, heating the power limiting reactor B.

f) After 7 minutes, the tie line between Fisk Street B and Quarry Street, that is, the power limiting reactor B, was opened, and Quarry Street and Fisk A, came back to normal. About the same time, the 30,000 KW machine in Northwest Station began to lose its excitation, and was disconnected.

g) Fisk B and Northwest remained out of synchronism with each other, with practically zero voltage at the busbars, for six minutes longer. Then the voltage suddenly came back, the alternators in Fisk B pulling into synchronism with each other and with the one remaining (20,000 KW) machine in Northwest.

Remarks:

a) A short circuit at the busbars of a station section, pulling the voltage down to zero, necessarily must drop out all the synchronous machines on this section, unless the short circuit is opened so quickly, that the synchronous machines during the period of zero voltage did not yet drop behind by half a pole, but at the reappearance of voltage are still in step. With a dead short this would usually require opening the short in a fraction of a second.

b) The purpose of sectionalizing the system by power limiting reactors was to limit trouble occurring on one station section to this section, and keep it from affecting the other sections.

The system of sectionalizing therefore has not worked satisfactorily in this case, as all the synchronous machines in Northwest, and nearly

half in Fisk A and Quarry Street have dropped out due to the trouble in Fisk B.

There is no power limitation between the Northwest Station and Fisk B, and any trouble on one of these two stations thus affects the other with practically the same severity. This is undesirable, as thereby trouble on one of these stations affects too large a part of the entire system. It is dangerous, as Fisk B and Northwest combined give too large a power for safe handling under all emergencies. Furthermore, due to the connection between these stations being practically all resistance and no reactance, the synchronizing power between Fisk B and Northwest must be small, and when synchronism is once lost under short circuit, etc., trouble must be anticipated in these stations pulling into synchronism with each other.

The interference between Fisk A, Quarry Street and Fisk B sections—which are connected with each other by power limiting reactors—results in serious trouble on one of these sections dropping out numerous synchronous machines in the other sections. It may partly be direct interference between the station sections, partly through substations fed simultaneously by several of these sections, and either requires further investigation.

c) If the busbar voltage of a station section drops to zero by short circuit, for a sufficiently long time to permit the turbine speeds to change appreciably, this station section is out of synchronism with the rest of the system, as at short circuit of zero voltage there is nothing to hold it in step. As soon, however, as the short is removed, the voltage should come back and the station section drop into step again with the rest of the system. This did not occur, but station sections remained out of step with each other at practically zero voltage for a considerable time, about a quarter of an hour. Apparently, the synchronizing power between the station sections is lower than desirable, and the speed control of the alternators not such as to bring them promptly so close together in speed as to drop into step.

d) The tandem or chain connection of the stations has the disadvantage that if an intermediary station, as Fisk B or Quarry Street, even momentarily drops out of synchronism by a short circuit, the system is cut in two. Ring connection of the station sections would have the advantage that, if one station section drops out of step by some accident, all the other station sections are still connected with each other and thereby can remain in synchronism.

2.) Oct. 22nd, 1919—12:20 P. M.

The trouble was very similar to that on September 18th, 3:37 P. M., a short circuit close to the busbar of Fisk B, except that:

a) The short circuit apparently opened very quickly.

b) The tie lines were operated manually, separating Fisk B and Northwest stations.

c) Most of the synchronous machines on Fisk B and Northwest, and a few on Fisk A and Quarry Street dropped out: Of 52 synchronous machines on Fisk B and Northwest, 46 are recorded as dropped out, six as remaining in synchronism; of 6 machines on Quarry Street, 2, and of 35 machines on Fisk A, 4 dropped out, the remaining stayed in step.

d) In Fisk Street B, and Northwest, the voltage dropped to practically zero, but came back at the opening of the tie line.

Remarks:

Apparently, the interference between the stations was less in this case, probably due to the short duration of the short circuit, and the opening of the tie line B, so that only few synchronous machines fell out in the other sections, and some even stayed in step in the disturbed section if the report is correct.

3.) Sept. 18th, 1919—5:27 P. M.

a) One hour forty minutes after the first trouble, resulting from a short near the busbars of Fisk B, a short circuit occurred near the busbar of Fisk A, and held for several seconds, while the tie line reactor B was still open, that is, Fisk B and Northwest cut off from Quarry Street and Fisk A.

b) All synchronous machines on Fisk A dropped out, and a few on Quarry Street, Fisk B and Northwest; 39 synchronous machines on Fisk A are recorded as having dropped out, 3 on Quarry Street, 5 on Fisk B and 1 on Northwest.

Remarks:

a) Some synchronous machines dropped out on Fisk B and Northwest, although the tie line between these stations and the station in which the trouble occurred, was open, showing interference between generating stations through the substations; these synchronous machines were in the same stations with machines fed from Fisk A or Quarry Street.

b) The synchronous machines on Fisk B, Northwest and Quarry Street, which dropped out, did so by overload. This is undesirable as with numerous synchronous machines, fed from the troubled station A, dropped out, those which receive power from the other stations should hold on to carry the load.

4.) May 19th, 1919—7:25 A. M.

a) A generator in Fisk Street A burned out, short circuiting with only the generator power limiting reactance, of about .4 ohms, between the short and the busbars.

b) The voltage dropped about 1,000 volts in all four station sections—a little more in Fisk A, where the trouble occurred, a little less in Fisk B and Northwest, the stations most remote from the trouble.

c) A violent fluctuation of voltage resulted in all four stations, with an amplitude apparently of 1,000 to 2,000 volts, most severe in Fisk A, where the trouble originated, of an irregular period of about 1 second per beat.

d) The tie line reactor B, got very hot.

e) The drop of voltage and the voltage fluctuation lasted for 18 minutes, with only slight decrease. Then they suddenly disappeared and normal voltage returned.

f) During the disturbance, the frequency of the system fluctuated by about two cycles, that is 8%, and three machines in Fisk A—where the trouble originated—tripped their excess speed governors and cut off steam.

g) Some synchronous machines dropped out of step, but the exact record is no more available.

Remarks:

a) The fluctuation and drop of voltage, extending over all four stations, and its long duration, probably were of similar nature and cause as the loss of voltage in the trouble on September 18th and October 22nd.

b) The most serious question, which unfortunately cannot be decided with the present data, is: whether this voltage drop and fluctuation was an actual hunting of the generating stations against each other, due to lack of synchronizing power, or whether it was hunting of the steam governors of the stations against each other, or whether it was only apparent, and due to the reaction on the generators of the substations when starting synchronous machines. This should be further investigated.

c) It appears to me certain that in this case the generating stations have remained in synchronism with each other throughout the entire 18 minutes of disturbance. If the station sections had dropped out of synchronism with each other, materially greater voltage drops should have occurred.

d) The observed speed fluctuation, by 8%, then would mean that the entire system simultaneously speeded up and slowed down, but not that material speed differences existed at the same time between different turbines. During such periodic speed pulsations, temporarily some machines may run faster than others, but for such short time only, as not to slip out of synchronism. As the observed speed fluctuation is close to the range, for which the excess speed governors are set, it may be expected that some machines may trip their emergency cut off, and as the speed fluctuation may be expected to be greatest at the source of disturbance, Fisk A, the tripping out may be expected first in Fisk A, as was observed; 3 machines tripped out there.

e) Suppose a large number of synchronous machines are tripped out by a momentary short circuit, but the voltage immediately comes back at the opening of the short, and a number of substations immediately proceed to start their synchronous machines from the alternating side. Even with such large station sections, the starting of several large synchronous machines would temporarily pull down the voltage. The voltage would rapidly recover by the speeding up of the synchronous machines which had been started. This would cause some other substations to start their synchronous machines and again pull down the voltage, and so a series of successive voltage drops and recoveries would result in irregular sequence, until the last synchronous machine is started. This would, in the voltage curve, give the appearance of a hunting pulsation, such as shown by the records. This theoretically is a possibility, but whether it was the cause, cannot be decided from the records.

Synchronoscopes between the station sections, however, would indicate that it is not a true hunting, due to instability of the station sections, but a voltage fluctuation due to excessive fluctuating lagging load, as given by the starting of synchronous machines.

f) Sufficiently sensitive synchronoscopes between the station sections would indicate whether the station sections are in phase with each other or out of synchronism, whether they are hunting against each other, and whether and what phase displacement exists between the station sections.

III—OPERATION

Momentum of Alternators

The emergency steam cut offs of the turbo-alternators are stated to be set for an excess speed of about 10%.

Considering the 12,000 KW units typical and of most interest, since most of the units in Fisk Street where the trouble originated are of this size.

An increase of speed of 10% with the steam valve open for full load, would require 2 2/3 seconds. With the steam valve opened for overload, by a momentary short circuit, the speeding up to the speed limit would occur still more rapidly. The ordinary steam governor cannot well be made to act quicker than this, in cutting steam off completely, without danger of steam governor hunting interfering with the stability of the system; furthermore, all the steam in turbine and the passages beyond the valve would still accelerate it. Thus tripping of the excess speed emergency steam cut off may be expected whenever a serious short circuit on a station section drops out all the synchronous machines and thereby suddenly relieves the load.

Deceleration tests were made by the operating engineers on four turbo-alternators, with excitation and without excitation, by allowing the machines to speed up on the throttle, until the excess speed cut off tripped, and then observing the rate of slowing down.

The observed excess speed above normal, at which the emergency cut off operated, varied from 6.7% to 12% with the different machines, and varied by as much as 1.3% in successive tests of the same machine. With the steam valve wide open during acceleration, an appreciably higher excess speed may be expected.

The rate of slowing down, after the steam is cut off, varied from 4% to 13.4% per minute at no excitation and from 6.75% to 20.4% per minute with the field excited.

It follows herefrom: suppose by a short circuit lasting an appreciable time, the load is dropped, and the turbo-alternators speed up and trip their emergency steam valves and then begin to slow down. Then the individual turbo-alternators in the different station sections, and even in the same station section, may be expected to differ materially from each other in speed, probably as much as 5 to 10%.

With such great differences of speed, between the different machines, diffiulties in synchronizing, that is, in the machines pulling each other into step, may be expected, especially between different station sections.

SYNCHRONIZING

A

If two alternators, or groups of alternators, such as station sections, are connected together in synchronism, that is, at the same speed, but out of phase with each other, they tend to pull each other into phase. The machine which is ahead in phase, gives off power and thereby drops back, slows down, and the machine which is behind in phase, receives power and thereby runs ahead, speeds up, and as the result, when the two machines have come in phase with each other, the one which was ahead, is going slower, the one which was behind is going faster, and the former now drops behind, the latter runs ahead in phase, and the phase difference between the machine reverses, and the machines thus oscillate against each other, while the interchange current between the machines fluctuates between a maximum value at maximum phase difference, and zero or a minimum value when machines are in phase. If there were no damping effects, this oscillation would continue with constant amplitude. Due to the damping effects exerted mainly by the lag of the field flux behind the resultant field excitation (the armature reaction component of the synchronous impedance), the amplitude of the oscillation steadily diminishes, until the oscillation disappears. That is, the interchange current between two alternators which are in synchronism but out of phase, pulsates, with approximately constant frequency of the beat, but with an amplitude, which gradually decreases to nothing.

If the EMFs of the two machines are equal, then at the moment when the two machines are in phase, there is no resultant EMF, and thus no current, and when the machines are out of phase, the resultant EMF is approximately in quadrature with the EMF of either machine. If then the circuit between the two machines should contain only resistance but no reactance, the interchange current between the two machines would be in phase with the resultant EMF, thus in quadrature to the EMF of either machine, or a wattless current with regards to the EMFs of the machines, that is, there would be no power transfer between the machines, or no synchronizing power. If, however, in the circuit between the two machines the resistance is negligible compared with the reactance, the interchange current lags (approximately), 90 degrees behind the resultant EMF, thus is in phase with the EMF of the one machine, in opposition with that of the other machine, thus is an energy current consuming power from the one and delivering power to the other machine. Thus only the reactive component of the interchange current between two alternators which are in synchronism but

out of phase, transfers power between the alternators and is synchronizing current, while the energy component of the interchange current does not contribute to bring the machines into phase.

If the EMFs of the two machines are unequal, then upon the power transferring or synchronizing current due to the phase difference, superposes a reactive current which magnetizes the machine of lower voltage and thereby raises its voltage, and demagnetizes the machine of higher voltage and thereby lowers its voltage, but does not transfer power between the machines and does not appreciably change the phenomena of power transfer by the synchronizing current, so that here and in the following, where we are mainly interested in the magnitude of the effects, we may for simplicity assume equality of EMF of the machines.

B

If two alternators or groups of alternators such as station sections, are connected together out of synchronism, that is while differing from each other in frequency, they slowly slip past each other, and during each cycle of slip, or beat, a periodic energy transfer takes place, while the interchange current periodically rises and falls. During one-quarter the cycle of slip, or beat, the alternators are partly in phase with each other, that is, their EMFs are in the same direction. The slower machine then receives energy and accelerates, the faster machine gives off energy and slows down and the two machines thus are brought nearer to each other in speed, pulled towards synchronism. During the next quarter cycle of speed, however, the alternators are partly in opposition, that is their EMFs are in opposite directions. The faster machine then receives energy and speeds up, the slower machine gives off energy and slows down, and the two machines pull apart again, by the same amount by which they pulled together in the preceding quarter cycle of slip. Thus the machines can pull into step only if the energy transferred during one-quarter cycle of slip is sufficient to bring them into step, that is, is larger than the energy required to speed the momentum of the slower machine up to synchronous speed (or slow down the momentum of the faster machine to synchronism). This gives the maximum value of slip from synchronism, at which the machines will promptly pull into step, or the limits of synchronizing power.

C

If, however, the two machines, or station sections, are out of synchronism with each other by a greater speed difference than that from which they can pull each other into step in one-quarter cycle of slip,

and if the machine voltage were perfectly constant, then the machines would never come into synchronism, but would continue indefinitely to slip past each other, coming nearer together during the one-quarter cycle of slip where their voltages are in the same direction, and drifting apart again by the same amount during the next quarter cycle of slip, where their voltages are in opposition. In reality, however, the EMF of the machines is not constant, but must vary periodically with the frequency of the current fluctuations, that is, twice the frequency of the slip. The current in the circuit between the two alternators pulsates, between large values during the quarter cycle of slip, when the EMFs of the alternators are in opposition, and small values during the next quarter cycle of slip, when the EMFs are in the same direction, and with it varies the armature reaction. The phase difference between the resultant current, and the EMF of each alternator periodically changes from 0 to 360 degrees, during each cycle of slip. With the periodic fluctuation of the current and its phase angle, the armature reaction of the machine fluctuates, and thereby gives a periodic fluctuation of voltage. As, however, the armature reaction requires an appreciable time to develop, the voltage fluctuation is not in phase with the fluctuating current, but lags behind it, by an angle depending on the time required for the armature reaction to exert its magnetizing effect. The result thereof is that the power interchange between the two alternators is not entirely alternating with the frequency of the slip, that is, alternately accelerating each machine and then again slowing it down by the same amount, but has a constant though small component, which is positive, that is, accelerating in the slower, and negative, that is retarding, in the faster machine. This represents a continuous synchronizing power, which steadily pulls the machines together in speed, until their speed difference has become small enough to pull suddenly into step.

If thus two alternators, or station sections, are out of synchronism with each other by so great a frequency difference, that they cannot pull each other into step within one-quarter cycle of slip, then the alternators continue slipping past each other, with large fluctuating currents flowing between them. These current fluctuations—and the voltage fluctuations caused by them—do not decrease (as in hunting or in other synchronous oscillations), but remain practically constant in amplitude, but the fluctuations gradually become slower, that is, the frequency of beats decreases, while the machines pull nearer together in frequency, that is, their speed difference decreases, until the critical speed difference is reached, where the acceleration during one-quarter

cycle of slip brings the machine up to full synchronism. Then suddenly the machines drop into step with each other, the excess current vanishes and normal voltage comes back, after a short oscillation of constant frequency of beats, but rapidly decreasing amplitude.

Apparently, this is what happened in the trouble on September 18th, 1919.

D

If two alternators or groups of alternators such as station sections, are connected to each other through a reactance, and the driving power of each group of alternators equals its load (plus losses), then no current and no power flows over the dividing reactor, and both alternators are in phase. If now, with the same load, the driving power on one alternator is increased, on the other decreased, or with the same driving power, the load on the one is increased, on the other decreased, but the terminal voltage kept the same, then current and power flows over the dividing reactor, resulting in a phase displacement between the two alternators (or station sections). This phase displacement increases with the increase of power transfer. This gives a case where current and power is carried over a reactance from one circuit to another one, without any voltage drop. It is this, on which the use of limiting busbar reactors is based. The power transfer reaches a maximum at 90 degrees phase displacement; this gives the limits of synchronizing power, and any further increase of power transfer causes the two alternators to drop out of synchronism with each other.

Too large reactance between the alternators reduces the synchronizing power by limiting the synchronizing current; too small reactance may again reduce the maximum synchronizing power by lowering the EMF by the large voltage drop due to the large interchange currents. With a capacity of about 60,000 KW per station section and machines of the general characteristics of those involved (100% synchronous reactance, 12½% true reactance, in average), maximum synchronizing power would require a reactance between each station section and the rest of the system of a little less than one ohm, so that the proposed arrangement of ring connection of the stations through power limiting reactors of 1.75 ohms should give about the maximum synchronizing power.

* * * * *

Mathematical and numerical data pertaining to the preceding are given in the Appendix.

Appendix

APPENDIX

Synchronous Operation

A

Consider the case of two alternators or groups of alternators such as station sections, which are running in synchronism with each other, that is, have the same frequency f, but are connected together while out of phase with each other by angle 2ω. That is, the one alternator has the voltage phase $(\phi-\omega)$, the other the voltage phase $(\phi+\omega)$.

We may assume the alternators as of equal voltage, since a voltage difference superposes on the synchronizing energy current due to the phase difference, a reactive magnetizing current due to the voltage difference without materially changing the energy relations.

The *EMFs* of the two alternators then may be represented by:

$$\left.\begin{aligned} e_1 &= E \cos (\phi-\omega) \\ e_2 &= E \cos (\phi+\omega) \end{aligned}\right\} \qquad (1)$$

and the *resultant voltage* in the circuit between the alternators then is:

$$\begin{aligned} e &= e_1 - e_2 \\ &= E \cos \left\{ (\phi-\omega) - \cos (\phi+\omega) \right\} \\ &= 2E \sin \omega \sin \phi \end{aligned} \qquad (2)$$

and the *interchange current*wbeteen the alternators is:

$$i = \frac{2E}{z} \sin \omega \sin (\phi - a) \qquad (3)$$

where:

$$z = r^2 + x^2$$

is the impedance of the circuit between the two alternators, and the phase angle a is given by:

$$\tan a = \frac{x}{r}$$

and:

r = resistance

x = reactance

of the circuit between the alternators (including their internal resistances and reactances).

The *power* of one of the two alternators then is given by:

$$p_1 = e_1 i$$

$$= \frac{2E^2}{z} \sin \omega \sin (\phi - a) \cos (\phi - \omega)$$

$$= \frac{E^2}{z} \sin \omega \sin \left\{ (2\phi - a - \omega) + \sin (\omega - a) \right\}$$

$$= \frac{E^2}{z} \sin \omega \sin (2\phi - a - \omega) + \frac{E^2}{2z} \cos a - \frac{E^2}{2z} \cos (2\omega - a) \qquad (4)$$

The phase angle ω of the EMF is not constant, but pulsates with approximately constant low frequency, the frequency of the beat, and decreasing amplitude.

$\omega_0 = \omega_{00} \epsilon^{-at}$ = maximum value of the phase angle, then may approximately represent the—gradually decreasing—amplitude of the phase angle, where a = attenuation of the beat or oscillation, and

$$\omega = \omega_{00} \epsilon^{-at} \sin p\phi \qquad (5)$$

would approximately represent the instantaneous value of the phase angle ω where:

pf = frequency of the beat, or the periodic variation of the phase angle.

[In the derivation of equations (3) and (4), ω has been assumed as constant. As ω is not constant, but by (5) a function of ϕ, additional terms appear in equations (3) and (4). Since, however, the frequency of variation of ω is very low compared with the frequency of ϕ: p = a small quantity; these additional terms in (3) and (4) are small, and equations (3) and (4) are correct with sufficient approximation, especially in the present case, where we are essentially interested in the magnitude of the power relations.]

In equation (4), the first term:

$$p_1{}^1 = \frac{E^2}{z} \sin \omega \sin (2\phi - a - \omega)$$

is of double the frequency 2f. It thus does not represent energy transfer between the two alternators, but merely represents the energy storage and return, twice per cycle, occurring in any inductive circuit. It thus is of no further interest.

The second term

$$p''_1 = \frac{E^2}{2z} \cos a$$

gives, substituting $\cos\alpha = \frac{r}{z}$;

$$p''_1 = \frac{E^2 r}{2z^2} \quad \frac{E^2 g}{2}$$

where g is the conductance of the circuit. That is, this term is the energy loss in the conductance, that is, the resistance of the circuit, and thus also is of no further interest.

The third term:

$$p''_1{}' = -\frac{E^2}{2z}\cos(2\omega - \alpha)$$

is of the low frequency of the beat, or the current fluctuation between the two alternators: pf. It thus represents the energy transfer between the two alternators, during their periodic oscillations.

or, resolving the last equation:

$$p''_1{}' = -\frac{E^2}{2z}\sin\alpha\sin 2\omega - \frac{E^2}{2z}\cos\alpha\cos\omega.$$

The second term:

$$p''_1{}'' = -\frac{E^2}{2z}\cos\alpha\cos\omega$$

has the same sign for negative ω, that is, when the machine is lagging, as for positive ω when the machine is leading, thus it represents no energy transfer between the machines.

The *synchronizing power*, or energy transfer during the synchronizing oscillations of two alternators, which are out of phase but in synchronism, thus is given by the expression:

$$p = \frac{E^2}{2z}\sin\alpha\sin 2\omega \tag{6}$$

Thus, the synchronizing power p, is a maximum, and is:

$$p_0 = \frac{E^2}{2z}\sin 2\omega \tag{7}$$

for $\alpha = 90$ degrees, that is, if the resistance r of the circuit between the alternators is negligible compared with the reactance.

The synchronizing power $p = 0$ for $\alpha = 0$, that is, in the (theoretical) case, when the circuit between the alternators contains no reactance, but only resistance.

For phase angles ω up to 45 degrees, that is, phase displacements between the two alternators up to $2\omega = 90$ degrees, the synchronizing power increases; beyond this it decreases again and becomes zero for $2\omega = 180$ degrees phase displacement.

The average value of p may be approximated by:

$$\text{avg. p} = \frac{E^2}{2z} \sin \alpha \, \frac{2\,(1-\cos 2\omega_0)}{\pi\, \omega_0} = \frac{E^2}{\pi \omega z} \sin \alpha \, (1-\cos 2\omega_0)$$

where ω_0 denotes the maximum value of ω
and as the duration of one half cycle of oscillation—during which the power transfer remains in the same direction—is given by equation (5) as:

$$p\phi = 2\pi pft = \pi$$

that is,

$$t = \frac{1}{2pf}$$

The *energy transfer* between the two machines, during each half cycle of oscillation, is given by:

$$W = \frac{1}{2pf} \text{ avg. p.} \tag{8}$$

$$= \frac{E^2}{2\,\pi pf \omega_0 z} \sin \alpha \, (1-\cos 2\omega_0)$$

in this, E is the maximum value of the voltage of each machine. Denoting by:

$$E_0 = \frac{E}{\sqrt{2}}$$

the effective value of the machine voltage, gives the effective values:

Resultant voltage:

$$e^0 = 2\, E_0 \sin \omega \tag{2^1}$$

Current:

$$i_0 = \frac{2E_0}{z} \sin \omega \tag{3^1}$$

Power transfer:

$$p = \frac{{E_0}^2}{z} \sin \alpha \sin 2\omega \tag{6^1}$$

Energy transfer during each half cycle of oscillation or beat:

$$W = \frac{{E_0}^2}{\pi pf \omega_0 z} \sin \alpha \, (1-\cos 2\omega_0) \tag{8^1}$$

where:

$$\omega = \omega_{00} \epsilon^{-at} \sin p\phi \tag{5}$$

is the angle of phase displacement of either machine, from the average;

$z = \sqrt{r^2 + x^2}$

$t \text{ an } \alpha = \frac{x}{r}$

r = resistance of circuit.
x = reactance of circuit.
p = frequency ratio of beat or oscillation, that is,
pf = frequency of oscillation, and
E_0 = effective value of machine EMF.

As seen from the equations, during each complete cycle of the oscillations, of frequency pf, the current i twice rises and falls, thus reaching two maxima, and the power p twice reaches a maximum and twice reverses, so that the energy W flows one way during half the cycle, and in the opposite direction during the other half cycle. The frequency of the rise and fall of the current thus is 2 pf.

Figure 1.—I and II show the curve i and voltage e_1 of the oscillation, for the (exaggerated) value $p = .1$, for $\omega_0 = 45$ degrees and $\omega_0 = 90$ degrees.

B

Consider now the case that two alternators, or groups of alternators such as station sections, are connected together while different from each other in frequency by 2s, that is, one alternator has the frequency $(1-s)\,f$, the other the frequency $(1+s)\,f$.

We may again assume the alternators as of equal voltage, since a voltage difference merely superposes on the synchronizing energy current a reactive magnetizing current, without materially changing the energy relations.

The *EMFs* of the two alternators then may be represented by:

$$\left.\begin{aligned} e_1 &= E \cos (1-s)\,\phi \\ e_2 &= E \cos (1+s)\,\phi \end{aligned}\right\} \qquad (9)$$

The *resultant voltage* in the circuit between the two alternators then is:

$$\begin{aligned} e &= e_1 - e_2 \\ &= E\left\{\cos (1-s)\,\phi - \cos (1+s)\,\phi\right\} \\ &= 2E \sin s\phi \sin \phi \end{aligned} \qquad (10)$$

[Assuming now that s is a small quantity (just as we assumed in A, that p is a small quantity), that is, that the two alternators have nearly the same frequency. The change of sin $s\phi$ then is slow compared with that of $\sin\phi$, and for all phenomena of the frequency f, sin $s\phi$ may be assumed as constant, and the reactance of the circuit

may be assumed as the same, $x = 2\pi fL$, for both component EMFs, e_1 and e_2 that is, for both frequencies $(1-s)$ f and $(1+s)$ f.]

The *interchange current* between the alternators then is:

$$i = \frac{2E}{z} \sin s\phi \sin (\phi - \alpha)$$

where:

$$z = \sqrt{r^2 + x^2}$$

$$\tan \alpha = \frac{x}{r} \qquad (11)$$

With regards to the EMF of one of the alternators, for instance, e_1 this current always lags. Its lag is 90 degrees when the current is a maximum. With decrease of current, the lag decreases from 90 degrees in the one, and increases in the next beat, and approaches in phase respectively in opposition, when the current is a minimum. The power factor thus varies from zero at maximum current, to unity at zero current, and its average thus is low. Fig. 1 shows as Curve III the relation of e_1 to i for the exaggerated values $s = .09$.

The *power* of one of the two alternators then is given by:

$$\begin{aligned} p &= e_1 i \\ &= \frac{2E^2}{z} \sin s\phi \sin (\phi - \alpha) \cos (1-s)\,\phi \\ &= \frac{E^2}{z} \sin s\phi \left\{ \sin [(2-s)\,\phi - \alpha] + \sin (s\phi - \alpha) \right\} \\ &= \frac{E^2}{z} \sin \phi \sin [(2-s)\,\phi - \alpha] + \frac{E^2}{2z} \cos \alpha - \frac{E^2}{2z} \cos (2s\phi - \alpha) \end{aligned}$$

The first term again is the double frequency term representing the energy storage by inductance; the second term is the power consumed by the resistance of the circuit. Neither thus represents energy transfer between the alternators.

The third term:

$$p = -\frac{E^2}{2z} \cos (s\phi - \alpha) \qquad (12)$$

is a slow pulsation of energy, which alternately accelerates the machine and thus tends to bring it nearer to synchronism, and then retards it again.

Usually, it is approximately: $a = 90$ degrees, that is, the reactance is large compared with the resistance, and equation (12) then becomes:

$$p' = \frac{E^2}{2z} \sin 2s\phi$$

During each cycle of the frequency sf, of the slip from synchronism or average frequency, the amplitude of the current i thus twice becomes zero and in phase, and twice reaches a maximum, when the alternators are in opposition, and the power p four times reaches a maximum and four times becomes zero and reverses, twice when the current comes into phase with the EMF, but the current becomes zero, and twice when the current is a maximum, but in quadrature with the EMF, and the power thus becomes zero. The power transfer between the alternators thus reverses four times per complete cycle of slip, sf, that is, is of the frequency 2sf, with two positive and two negative maxima.

The average value of the power is:

$$\frac{2}{\pi} p = \frac{E^2}{\pi z}$$

and as the duration of one quarter cycle of slip is $t = \frac{1}{4sf}$, the *energy transfer* between the two machines, during a quarter cycle of slip thus is:

$$W = \frac{1}{4sf} \frac{2}{\pi} p$$

$$= \frac{E^2}{4\pi sfz} \qquad (13)$$

There is thus that difference between the slipping of alternators past each other out of synchronism, and the oscillation of the alternators against each other at synchronism (A), that in the slipping the power fluctuation and the reversal of the energy is of twice the frequency of the current fluctuation, while in the oscillation of the alternators against each other at synchronism, the power fluctuation or reversal of energy flow is of the same frequency as the current fluctuation.

If two alternators are connected together while out of synchronism, and slowly slip past each other, during each half cycle of slip, or beat, while the two machines EMF pass from in phase to in opposition to in phase again, a periodic energy transfer takes place. During one quarter cycle of slip (that is, while one alternator EMF slips behind, the other pulls ahead of the minimum frequency by one quarter cycle, and the two alternator EMFs thus slip against

each other by one quarter cycle), the alternators are partly in phase with each other, and the slower machine receives energy from the faster machine. The two machines are thereby brought nearer to each other in speed, pulled towards synchronism. During the next quarter cycle of slip, however, the two alternators are partly in opposition, and the faster machine receives energy from the slower one. The faster machine then speeds up, the slower machine slows down, and the two machines pull apart again, by the same amount by which they pulled together in the preceding quarter cycle of slip (if their EMF is constant). Thus the two machines can pull into step only if the energy transferred during one quarter cycle of slip, W, is larger than the energy required to speed up the momentum M of the machine, to full synchronism.

Due to the energy transfer W between the machines, resulting in an alternate speeding up and slowing down, the slip s is not constant, but pulsates periodically, between the minimum value $s-s_1$, at the end of the quarter cycle during which the machines pull together, and beginning of the quarter cycle during which the machines pull apart, and a maximum value $s+s_1$, at the end of the quarter cycle, during which the machines pull apart, and beginning at the quarter cycle during which the machines pull together—where s_1 is the amplitude of the pulsation of slip. As the energy required to accelerate the momentum M of the machine by the speed $2s_1$ is $4s_1M$, it follows:

$$W = 4s_1M$$

or: $$s_1 = \frac{W}{4M}$$

$$s_1 = \frac{E^2}{16\,\pi sfzM} \tag{14}$$

is the amplitude of the speed fluctuation of the two alternators during their slipping past each other, out of synchronism with the slip s.

$s_1 = s$ gives the minimum slip $s - s_1 = 0$, that is, the machines pull into synchronism.

The maximum slip s_1 from which the two machines pull into synchronism with each other, thus is given by substituting $s_1 = s$ in (14), as:

$$s_0 = \frac{E}{\sqrt{4\,\pi fzM}} \tag{15}$$

s_0 thus is the *limit of synchronizing power.*

Substituting again the effective value of EMF, E_0, for the maximum value E, by:

$$E_0 = \frac{E}{\sqrt{2}}$$

gives the effective values:

Resultant voltage:

$$e^0 = 2E_0 \sin s\phi \tag{10¹}$$

Current:

$$i_0 = \frac{2E_0}{z} \sin s\phi \tag{11¹}$$

Power transfer:

$$p = \frac{E_0^2}{z} \cos (2s\phi - a) \tag{12¹}$$

Energy transfer during one quarter cycle of slip:

$$W = \frac{E_0^2}{2\pi sfz} \tag{13¹}$$

Amplitude of pulsation of slip:

$$s_1 = \frac{E_0^2}{8\,\pi sfzM} \tag{14¹}$$

Critical Slip, from which the machines pull each other into synchronism, or limits of synchronizing power:

$$s_0 = \frac{E_0}{2\sqrt{2\pi fzM}} \tag{15¹}$$

These expressions are very similar to those of A, with $s\phi$ taking the place of ω, and 2s the place of p.

C

With two machines out of synchronism with each other by a greater speed difference 2s, than that, from which the machines can pull each other into synchronism within one quarter cycle of slip, from the equations of B it would follow, that the machines can never pull each other into synchronism, if the voltage E_0 is constant, but must indefinitely continue to slip past each other, coming nearer together during one quarter cycle of slip, and dropping apart again by the same amount during the next quarter cycle of slip.

This, however, is under the assumption, that the machine EMF E_0 is constant. In reality, however, E_0 is not constant, but varies periodically with the same frequency as the current fluctuates. The

current in the circuit between the machines and thus the armature reaction in the machine varies in amplitude and in phase difference against the machine voltage, and the machine voltage varies with the amplitude and the phase of the armature reaction.

Consider, as approximation, the armature reaction as proportional to the quadrature component of the current. The EMF of the machine would then be expressed by an approximate equation of the form:

$$E = E_0 \left\{ 1 - c \sin s\phi \sin \varphi \right\}$$

where φ is the phase angle between the current and the EMF and $\sin s\phi$ represents the amplitude of the current pulsation, by (11^1)

it is, however, by (9) and (11):

$$\varphi = (\phi - \alpha) - (1 - s)\,\phi - 90 \text{ degrees}$$
$$= s\phi - \alpha + 90 \text{ degrees}$$

thus:

$$E = E_0 \left\{ 1 + c \sin s\phi \cos (s\phi - \alpha) \right\} \tag{16}$$

Substituting (16) into the expression of the power of the alternator (12^1), the equations still remain alternating, that is, there is no resultant synchronizing power, but equal positive and negative values of power alternate.

However, (16) assumes that the magnetizing effect of the armature reaction is instantaneous, that is, that the EMF E at any moment is the value corresponding to the armature reaction existing at this moment. This, however, is not the case, and the armature reaction is not instantaneous, but requires an appreciable time, several seconds, to develop, and the magnetizing or demagnetizing effect of the armature reaction on the voltage therefore materially lags behind the armature reaction.

Let then σ = angle of lag of the voltage change behind the armature reaction which causes it. It is then—

$$E = E_0 \left\{ 1 + c \sin s\phi \cos (s\phi - \alpha - \sigma) \right\} \tag{17}$$

and, substituting (17) into (12^1), give the *power transfer* between the machines:

$$p = \frac{E_0^2}{z} \cos (2s\phi = \alpha) \left\{ 1 + c \sin s\phi \cos (s\phi - \alpha - \sigma) \right\}^2$$

or approximately, since c is a small quantity:

$$p=\frac{E_0^2}{z}\cos(2s\phi-a)+\frac{2cE_0^2}{z}\cos(2s\phi-a)\sin s\phi\cos(s\phi-a-\sigma)$$

The first term:

$$p'=\frac{E_0^2}{z}\cos(2s\phi-a)$$

is the slowly alternating energy transfer between the machines which causes their speed to fluctuate, but does not permanently bring them nearer to each other, that is, exerts no synchronizing power.

The second term:

$$p''=\frac{2cE_0^2}{z}\cos(2s\phi-a)\sin s\phi\cos(s\phi-a-\sigma)$$

$$=\frac{cE_0^2}{z}\cos(2s\phi-a)\left\{\sin(a+\sigma)+\sin(2s\phi-a-\sigma)\right\}$$

$$=\frac{cE_0^2}{z}\sin(a+\sigma)\cos(2s\phi-a)+\frac{cE_0^2}{2z}\left\{\sin(4s\phi-2a-\sigma)-\sin\sigma\right\}$$

The first two terms of this expression:

$$\frac{cE_0^2}{z}\sin(a+\sigma)\cos(2s\phi-a)$$

and

$$\frac{cE_0^2}{2z}\sin(4s\phi-2a-\sigma)$$

are slowly alternating, thus represent no permanent acceleration, that is, no synchronizing power.

The third term however:

$$p_0\frac{cE_0^2}{2z}\sin\sigma \tag{18}$$

is constant, that is, represents a continuous synchronizing power, which steadily pulls the machines together, until their speed difference 2s has become small enough to pull the machines suddenly into step.

If thus two alternators or station sections, are considerably out of synchronism with each other, they continue slipping past each other, with large fluctuating currents flowing between them, and the speeds of the machines fluctuating with the fluctuation of the current. These currents do not decrease in amplitude, but remain of practically

constant value, but their period of fluctuation gradually gets slower, that is, the fluctuation gradually becomes slower, while currents slowly pull the machines nearer into synchronism with each other, that is decrease their frequency difference, until the critical frequency $2s_0$ is reached (where the acceleration during a quarter cycle of slip, $2s_1$, reaches full synchronism.) Then the machines suddenly drop into synchronism, but oscillate in phase against each other, with an approximately constant frequency of oscillation, but with a current fluctuation, which steadily (and usually rapidly) decreases,until steady conditions of speed, current and voltage are reached.

The armature reaction of the alternator is represented by the difference of the synchronism reactance x_0 and the true reactance x_1, that is, by an effective *reactance of armature reaction.*

$x_2 = x_0 - x_1$.

The co-efficient c in the synchronizing power, p_0 (18), is that fraction of the reactance of the armature reaction x_2, which appears during the short time of the current fluctuation. Thus c is the larger, the slower the fluctuation, that is, the less s. In other words, c increases with decreasing slip, that is, increasing approach to synchronism.

Inversely, σ is a maximum and practically 90 degrees for large values of s, where the voltage fluctuation lags practically 90 degrees behind the fluctuation of the armature reaction, and decreases with decreasing s, that is, increasing approach to synchronism, c sin σ and thus the synchronizing power p_0 (18), thus should be a maximum at some moderate slip s, and decrease for larger as well as smaller slips.

Assuming that it takes t_0 seconds for the field to build up to correspond to the armature reaction. With the current fluctuating with the frequency 2sf, and assuming that the magnetizing effect of the armature reaction is sinoidal which at best is but a very crude approximation, it would be:

$$c = \frac{1}{4\,sft_0}$$

and:

$$\sin\sigma = \sqrt{1 - \left(\frac{1}{4\,sft_0}\right)^2}$$

thus:

$$p_0 = \frac{E_0^2}{8zsft_0}\sqrt{1 - \left(\frac{1}{4\,sft_0}\right)^2} \qquad (19)$$

However, secondary effects occur and more or less modify the value p_0 such as the effect of secondary currents, induced in the field structure by that component of the armature current which is due to the EMF of the other machine, and which gives an induction motor torque, tending to pull the machines together into synchronism.

D

Consider two alternators or groups of alternators such as station sections of the same terminal voltage, connected with each other through an impedance z, and in synchronism with each other. If then the load distribution between the alternators differs from the distribution of their driving power, electric power is transferred over the impedance z, current flows and a phase displacement 2ω occurs between the two sides of the reactor z.

In this case, the phase angle ω is constant, and not periodically fluctuating as in A, but varies with changes of distribution of load; the equations, however, are the same as in A, except that now ω is constant, and the voltages e_1 and e_2 are the terminal voltages.

It thus is,

Current in the impedance:

$$i=\frac{2E}{z}\sin\omega\sin(\phi-\alpha)$$

Voltage across the impedance:

$$e=2E\sin\omega\sin\phi$$

Power transferred over impedance:

$$p=\frac{E^2}{2z}\left[\cos\alpha-\cos(2\omega-\alpha)\right]$$

where E is maximum value of the terminal voltages:

$$e_1=E\cos(\phi-\omega)$$
$$e_2=E\cos(\phi+\omega)$$

and α the phase angle of the impedance.

If the impedance z is a reactance, that is, the resistance r, is negligible: $r=0$, it is $\alpha=90$ degrees, and:

$$i=-\frac{2E}{x}\sin\omega\cos\phi$$

$$e=2E\sin\omega\sin\phi$$

$$p=\frac{E^2}{2x}\sin 2\omega$$

or, substituting the effective values: $E = E_0 \sqrt{2}$:

$$i_0 = \frac{2E_0}{x} \sin \omega$$

$$e^0 = 2E_0 \sin \omega$$

$$p = \frac{E_0^2}{x} \sin 2\omega$$

The power p thus is zero for $\omega = 0$, increases, reaches a maximum:

$$p_m = \frac{E_0^2}{x}$$

for $\omega = 45$ degrees, or 90 degrees phase displacement between the lternators, and then decreases again to zero at $\omega = 90$ degrees, or opposition.

At maximum power transfer, it is:

$$i_m = \frac{E_0 \sqrt{2}}{x}$$

$$e_m = E_0 \sqrt{2}$$

$$p_m = \frac{E_0^2}{x}$$

E

The foremost difficulty, and uncertainty in the application of the preceding equations, is found in the selection of the proper values of the machine EMF E_0. E_0 is not the terminal voltage; by slipping past each other without external impedance, the terminal voltage of the alternators goes down to zero. Neither is E_0 the "nominal induced voltage," as this has no actual existence, but is the voltage which would be induced by the field excitation if the saturation curve of the machine continued as a straight line. It appears to me that E_0 must be considered as the actual induced voltage, that is, the voltage induced by the actual field flux, that is, the field flux due to the resultant of field excitation and armature reaction. The armature reaction, however, fluctuates with the current between zero and a maximum, while the actual field flux is practically constant, since the magnetic field cannot follow the relatively rapid fluctuation of armature reaction.

The magnetic effect of the armature reaction is represented electrically in the synchronous reactance x_0. The synchronous reactance thus consists of a true self-inductive reactance x_1, which is instantaneous, and an effective reactance of armature reaction x_2,

which requires appreciable time to develop, and does not correspond to any real magnetic flux.

$x = x_1 + x_2$

In turbo-alternators, x_2 usually is very much larger than x_1.

Electrically, the actual induced EMF thus should be the nominal induced voltage e_0, which corresponds to the field excitation, less the reactance drop of the average current in the effective reactance of armature reaction, x_2.

If then I = maximum effective value of the fluctuating current, the average current is $\frac{I}{2}$, and the actual induced voltage thus is:

$$E_0 = e_0 - \frac{Ix_2}{2}$$

It is, however, in two alternators connected together out of synchronism, through an additional reactance:

$$2\ E_0 = I\ (2x_1 + x)$$

where x is the additional reactance through which the alternators of actual induced voltage E_0 and true self-inductive reactance x_1 are connected together, while running out of synchronism with each other.

From these two equations follows:

Maximum (effective) value of the fluctuating interchange current:

$$I = \frac{2e_0}{2x_1 + x_2 + x} \tag{20}$$

and, actual induced voltage:

$$E_0 = e_0 \frac{2x_1 + x}{2x_1 + x_2 + x} \tag{21}$$

where e_0 = nominal induced voltage, effective value.

If the alternators are connected through an impedance z, z takes the place of x, combining vectorially with x_1 and x_2.

In this calculation, the armature reaction has been assumed as demagnetizing, and the impedance voltage therefore subtracted from the nominal induced voltage. This appears correct, as the interchange current between the alternators out of synchronism with each other, is essentially a lagging current, throughout, as illustrated in Fig. 1.

If the two alternators are in synchronism, but out of phase with each other by a maximum phase displacement angle $2\omega_0$, it is:

$$2E_0 \sin \omega_0 = I\ (2x_1 + x)$$

and, again assuming the armature reaction as demagnetizing:

$$E_0 = e_0 - \frac{Ix_2}{2}$$

thus: the maximum (effective) value of the fluctuating exchange current:

$$I = \frac{2e_0 \sin \omega_0}{2x_1 + x + x_2 \sin \omega_0} \tag{22}$$

and, actual induced voltage:

$$E_0 = e_0 \frac{2x_1 + x}{2x_1 + x + x_2 \omega_0} \tag{23}$$

where e_0 is the nominal induced voltage, effective value.

However, in this case of alternators in synchronism but oscillating against each other, at least for small and moderate values of ω_0 the interchange current I is essentially an energy current with regards to the machine voltage, and the reactive component of this current alternately changes between lag and lead, that is, between demagnetizing and magnetizing. Therefore, the correctness is doubtful of subtracting the impedance voltage from the nominal induced voltage to get the induced voltage, but it would be:

$$E_0 = \sqrt{e_0^2 - i^2 x_2^2}$$

and as i varies between 0 and I, the average of E_0 would be the mean between e_0 and $\sqrt{e_0^2 - I^2 x_2^2}$, thus:
combining with the equation:

$$2E_0 \sin \omega_0 = I\ (2x_1 + x)$$

gives:

$$I = \frac{2e_0\ (2x_1 + x) \sin \omega_0}{(2x_1 + x)^2 + x_2^2 \sin^2 \omega_0} \tag{24}$$

$$E_0 = e_0 \frac{(2x_1 + x)^2}{(2x_1 + x)^2 + x_2^2 \sin^2 \omega_0} \tag{25}$$

It is probable that the true value of E_0 lies between (23) and (25), but nearer to (25).

Substituting these values (21), (23), (25) into the equations of A, B and C, and substituting

$$z = 2x_1 + x$$

in these equations, as the impedance of the circuit between the two alternators, gives the equations tabulated in *F*.

The nominal induced EMF e_0 is derived by combining the terminal voltage e with the impedance voltage iz, where z is the total

impedance inside of the terminals, true reactance as well as effective reactance of armature reaction. For non-inductive load—and synchronous machine load may be assumed as approximately non-inductive—this gives:

$$e_0 = \sqrt{e^2 + (ix)^2}$$
$$= e\sqrt{1+\xi^2} \qquad (26)$$

where ξ is the percentage reactance, and the resistance is neglected, as small compared with the reactance.

However, this expression neglects the change of reactance with increase of magnetic saturation, increase of magnetic leakage between field poles, etc., and therefore, especially in turbo-alternators with their enormous magnetic fields, high saturation and high field leakage, this expression is not very accurate, and reasonably reliable only in the mean range of current and voltage.

In B, and C, the case of two alternators, or groups of alternators out of synchronism with each other, the equations of synchronizing power, energy and critical slip: p, p_0, W, s_0, contain the term

$$\frac{2x_1 + x}{2x_1 + x + x_2}$$

thus are a maximum, if this term is a maximum. This is the case if:

$$x_2 = 2x_1 + x, \text{ or}$$
$$x = x_2 - 2x_1$$

that is:

The synchronizng power between alternators out of synchronism with each other is a maximum, and the frequency difference from which they pull each other into synchronism, is greatest, if the alternators or group of alternators are connected together through a reactance which is equal to the effective reactance of armature reaction, less twice the self inductive reactance of the circuit between the alternators or groups of alternators. With two alternators or groups of alternators connected together without any external reactance, this means, if the self inductive reactance of the alternators or group of alternators is one-third the synchronous reactance. With turbo-alternators, the self inductive reactance usually is much less, and with such machines the synchronizing power thus is increased by the insertion of external reactance.

Substituting above relation into the equations of B, and C, gives as the expressions for the case of maximum synchronizing power:

Actual machine EMF:	$E_0=\frac{e_0}{2}$
Resultant EMF:	$e^0=e_0 \sin s\phi$
Resultant current:	$i_0=\frac{e_0 \sin s\phi}{x_2}$
Power fluctuation of low frequency:	$p=\frac{e_0^2 \cos(2s\phi-\alpha)}{4x_2}$
Energy transfer of low frequency:	$W=\frac{e_0^2}{8\pi sfx_2}$
Continuous power transfer:	$P_0=\frac{ce_0^2 \sin\sigma}{8x_2}$
Critical slip:	$s_0=\frac{s_0}{4\sqrt{2\pi fx_2M}}$

F—Equations

A: Two alternators in synchronism, but out of phase with each other. Oscillation Frequency: pf.

B and C: Two alternators out of synchronism, slipping past each other. Frequency of slip: sf.

$$z = 2x_1 + x$$

Actual Machine E. M. F.

A:

$$E_0 = e_0 \frac{z}{z + x_2 \sin \omega_0} = e_0 \frac{2x_1 + x}{2x_1 + x + x_2 \sin \omega_0} = e_0 \frac{z^2}{z^2 + x_2^2 \sin^2 \omega_0}$$

$$= e_0 \frac{(2x_1 + x)^2}{(2x_1 + x)^2 + x_2^2 \sin^2 \omega_0}$$

B and C:

$$E_0 = e_0 \frac{z}{z + x_2} = e_0 \frac{2x_1 + x}{2x_1 + x + x_2}$$

Resultant E. M. F.

A:

$$e^0 = 2E_0 \sin \omega = \frac{2e_0(2x_1 + x)\sin \omega}{2x_1 + x + x_2 \sin \omega_0} = \frac{2e_0(2x_1 + x)^2 \sin \omega}{(2x_1 + x)^2 + x_2^2 \sin^2 \omega_0}$$

B and C:

$$e^0 = 2E_0 \sin s\phi = \frac{2e_0(2x_1 + x)\sin s\phi}{2x_1 + x + x_2}$$

Resultant Current

A:

$$i_0 = \frac{e^0}{z} = \frac{2e_0 \sin \omega}{2x_1 + x + x_2 \sin \omega_0} = \frac{2e_0(2x_1 + x)\sin \omega}{(2x_1 + x)^2 + x_2^2 \sin^2 \omega_0}$$

B and C:

$$i_0 = \frac{e^0}{z} = \frac{2E_0 \sin s\phi}{z} = \frac{2e_0 \sin s\phi}{2x_1 + x + x_2}$$

Power Fluctuation of Low Frequency

$$p=\frac{E_0^2}{z}\sin a \sin 2\omega=\frac{e_0^2(2x_1+x)\sin a \sin 2\omega}{(2x_1+x+x_2 \sin\omega_0)^2}=e_0^2\frac{(2x_1+x)^3\sin a \sin 2\omega}{[(2x_1+x)^2+x_2^2\sin^2\omega_0]^2}$$

$$\frac{E_0^2}{z}\cos(2s\phi-a)=\frac{e^2{}_0(2x_1+x)\cos(2s\phi-a)}{(2x_1+x+x_2)^2}$$

Energy Transfer of Low Frequency

$$W=\frac{E_0^2}{z}\sin a\ \frac{1-\cos 2\omega_0}{\pi p f\omega_0}=\frac{e_0^2(2x_1+x)\sin a\ (1-\cos 2\omega_0)}{\pi p f\omega_0(2x_1+x+x^2\sin\omega_0)^2}$$

$$=e_0^2\frac{(2x_1+x)^3\sin\ (1-\cos 2\omega_0)}{\pi p f\omega_0[(2x_1+x)^2+x_2^2\sin^2\omega_0]^2}$$

$$W=\frac{E_0^2}{2\pi sfz}=\frac{e_0^2(2x_1+x)}{2\pi sf(2x_1+x+x_2)^2}$$

Continuous Power Transfer

$p_0=$

$$P_0=\frac{cE_0^2\sin\sigma}{2z}=\frac{ce_0^2(2x_1+x)\sin\sigma}{2\ (2x_1+x+x_2)^2}$$

Attenuation

$$\omega=\omega_0\sin p\phi=\omega_{00}\epsilon^{-at}\ p\phi$$

$$\omega_0=\omega_{00}\epsilon^{-at}$$

Critical Slip

$s_0 =$

$$s_0 = \frac{E_0}{2\sqrt{2\pi fzM}} = \frac{e_0(2x_1+x)}{2\sqrt{2\pi fM}\ (2x_1+x+x_2)}$$

Pulsation of Slip

$s_1 =$

$$s_1 = \frac{E_0^2}{8\pi sfzM} = \frac{e_0^2(2x_1+x)}{8\pi\ sfM\ (2x_1+x+x_2)^2}$$

Pulsation of Armature Reaction

$$c = \frac{1}{4sft_0}$$

Lag of Armature Reaction

$$\sin\sigma = \sqrt{1-c^2}$$

Terminal Voltage of Alternator

$$E_1 = \frac{x+z}{2z} E_0 = \frac{x_1+x}{2x_1+x} E_0 = e_0 \frac{x_1+x}{2x_1+x+x_2 \sin\omega_0} = e_0 \frac{(x_1+x)\ (2x_1+x)}{(2x_1+x)^2+x_2^2 \sin^2\omega_0}$$

$$E_1 = \frac{x+z}{2z} E_0 = e_0 \frac{x_1+x}{2x_1+x+x_2}$$

Denotations

e_0 = nominal induced E. M. F. of alternator or group of alternators.

x_{11} = true self inductive reactance of alternator or group of alternators.

x_{12} = external reactance of alternator or group of alternators, thus.

$x_1 = x_{11} + x_{12}$ = total self inductive reactance of alternators or group of alternators.

x_2 = effective reactance of armature reaction of alternator or group of alternators, thus:

$x_0 = x_{11} + x_2$—synchronous reactance of alternator or group of alternators.

x = reactance (or impedance) between alternators.

z = impedance of circuit between alternators.
$= \sqrt{r^2 + (2x_1 + x)^2}$, where

r = resistance of circuit between alternators. Or approximately
$z = 2x_1 + x$

α = phase angle of circuit between alternators, where:

$\tan \alpha = \frac{2x_1 + x}{r}$ Or approximately:

$\alpha = 90$ degrees.

ω = phase displacement from mean, of oscillating alternators, thus:

2ω = total phase displacement of oscillating alternators from each other.

ω_0 = maximum phase displacement during oscillation.

ω_{00} = initial value of ω_0

pf = frequency of oscillation.

s = slip of frequency from mean, thus:

$2s$ = slip of frequency of alternators from each other, and

sf = frequency of slip from synchronism;

$2sf$ = frequency of slip of alternators from each other.

f = synchronous frequency.

t_0 = time required for magnetizing effect of armature reaction.

M = momentum, in joules, of alternator or group of alternators, per phase, that is:

$3M$ = total momentum.

G

The Constants of the Turbo-Alternators Used by the Commonwealth Edison Company are, as given by the Designing Engineers and the Operating Engineers:

Rating KW	Speed Rev.	Amps. at 9000 V.	Momentum 3M 10^6	Regulation %	Synchronous Impedance %	Internal Reactance: %	External Reactance: %	Reactance in Ohms: x_0:	x_{11}:	x_{12}:	x_1:	x_2
12,000	750	770	150	21	92	6¾	6	6.22	.458	.405	.863	5.76
14,000	750	900	200	22	85	5½	6	4.92	.318	.347	.665	4.60
20,000	750	1282	233	29	130	8	4.13	5.28	.325	.168	.493	4.95
25,000[1])		1600	255		>100		0				.45	4.00
30,000	1500	1925	262	24	119	8	6¼	3.20	.214	.168	.383	2.99
35,000	1500	2250	204	28	152	10		3.52	.232		.30[1])	3.29
Avg. 10,000[2])		640	120		100	6½	6	8.13	.53	.49	1.02	7.60

[1]). Estimated.

[2]). With allowance for the number of each type. Per 10,000 KW rating.

Busbar reactor: $x=1.75$.

Six tie cables between Fisk B and Northwest Station: $r=.312$.
$x=.074$.

H

In the trouble of September 18th, 1919, the following machines were involved:

In Fisk B and Northwest Station:

123,000 KW rating, 105,000 KW load = 85.5%

In Fisk A and Quarry Street:

114,000 KW rating, 90,000 KW load = 79%

It was:

Fisk A:

Six 12,000 KW; 98% reactance. 79% load. Thus:

$\sqrt{1+.79x.98^2} = 1.265$

$e_0 = 1.265 \times 5200 = 6600V.$ $x_2 = 5.76;$ $x_1 = .458 + .405 = 863;$ $M = 50 \times 10^6.$

Quarry St.:

Three 14,000 KW: 91% reactance. 79% load. Thus:

$\sqrt{1+.79 \times .91^2} = 1.23.$

$e_0 = 6400V.$ $x_2 = 4.60:$ $x_1 = .318 + .347 = .665;$ $M = 67 \times 10^6.$

Fisk B:

Four 12,000 KW; 98% reactance. 85.5% load. Thus:

$\sqrt{1+.855^2 \times 98^2} = 1.305.$

$e_0 = 6770V.$ One 25,000 KW was immediately shut down.

Northwest Station:

One 20,000 KW: 134% reactance, and one 30,000 KW: 125% reactance.

85.5% load.

20,000 KW: $\sqrt{1+.855^2 \times 1.34^2} = 1.52.$ $e_0 = 7900$ V.

$x_2 = 4.95;$ $x_1 = .325 + .168 = .493;$ $M = 78 \times 10^6.$

30,000 KW: $\sqrt{1+.855^2 \times 1.25^2} = 1.46:$ $e_0 = 7600$ V.

$x_2 = 2.99:$ $x_1 = .214 + .168 = .382:$ $M = 87 \times 10^6.$

Busbar reactors: $x = 1.75$

Tie cables $x = .074$ $r = .312.$

(1.) Assuming, at first, that the two machines in Northwest Station are in sychronism with each other, and the four machines in Fisk B are in synchronism with each other, but that Fisk B is out of synchronism with Northwest Station. It is then, in the circuit between Fisk B and Northwest Station:

$x_2 = 1.66;$ $x_1 = .074;$ $r = .312;$ $z = .594;$ $M_1 = 165 \times 10^6;$ $M_2 = 200 \times 10^6.$

Nominal induced EMF, average:
$e_0 = 7250$ V.

Actual induced EMF:
$E_2 = .263\ e_0 = 1910$ V.

Terminal voltage, average:
$E_t = .77\ E_0 = 1470$ V. $E_t\ \sqrt{3} = 2550$ V.

Current between stations, maximum effective value:
$i_0 = 6400$ A.

Max. pulsating power, per phase:
$p = 6150$ KW.

Max. steady power transfer, per phase:
$p_0 = 770$ KW.

Critical slip:
$s_0 = .77\%$; $s_0^1 = .70\%$; $s_0 + s_0^1 = 1.47\%$

These values do not well agree with the observations recorded, and while so many assumptions had to be made in the calculations that the exact numerical values can not be relied upon too closely, nevertheless, the general magnitude of the numerical values can generally be relied upon. It is probable that a terminal voltage of 2550 would not have escaped notice in the indicating meter; a slowly pulsating power transfer of $3p = 18,450$ KW would have shown on the record as a bad fluctuation, while the record shows an apparently steady flow of about 1000 KW only. Also, the fluctuating interchange current between the stations, rising to 6400 amperes, would probably have shown marked distress in the cable.

It must therefore be assumed that the generators in the same stations did not stay in synchronism with each other, and more particularly that the 20,000 and the 30,000 KW unit in Northwest Station broke out of synchronism. The behaviour of these machines, as the gradual loss of the field, is an indication in the same direction.

(2.) With the 20,000 and the 30,000 KW units of Northwest Station out of synchronism with each other on the busbars the following relations would be obtained:

$x_2 = 3.97$; $2x_1 = .875 = z$; $M_1 = 78 \times 10^6$; $M2 = 87 \times 10^6$.
$e_0 = 7750$ V.
$E_0 = .181\ e_0 = 1400$ V.
$i_0 = 3200$ A.
$p = 2230$ KW.
$s_0 = .68\%$ $s_0^1 = .64\%$ $s_0 + x_0^1 = 1.32\%$

Thus with the breaking of the synchronism between these machines, the voltage and the circulating current materially dropped, and with it also the current circulating between these machines and Fisk B.

The 30,000 KW machine then was shut down, leaving the 20,000 KW as the only unit in Northwest Station, drafting out of synchronism with Fisk B. Assuming, however, that the four 12,000 KW units in Fisk B have kept in synchronism with each other, the relations obtained:

(3.) One 20,000 KW unit in Northwest Station; four 12,000 KW units in Fisk B in synchronism with each other, but the two stations out of synchronism with each other:

$x_2=3.20$: $2x_1=.709$: $x=.074$: $r=.312$: $z=.84$: $M_1=78 \times 10^6$: $M_2=200 \times 10^6$.

$e_0=7330$ V.

$E_0=.208\ e_0=1520$ V.

$E_t=.69\ E_0=1050$ V. $E_t\ \sqrt{3}=1820$ V.

$i_0=3600$ A.

$p=2750$ KW.

$s_0=.75\%$: $s_0^1=.47\%$ $s_0+s_0^1=1.22\%$.

While the values of p and i_0 are much lower than in (1), still they are much larger than indicated by the records; 3600 A. maximum effective value of interchange current would give a loss in resistance of the tie cables of $\frac{3\ ri_0^2}{2}=600$ KW, in addition to the fluctuating power reaching a maximum of $3p=8250$ KW.

Assuming then that not only the two stations, Fisk B and Northwest, but also the individual generators in either station had broken synchronism. The conditions then pertain:

(4.) Two 12,000 KW alternators in Fisk B out of synchronism with each other:

$x_2=5.76$: $2x_1=1.726=z$: $M=50 \times 10^6$.

$e_0=6770$ V.

$E_0=.23\ e_0=1560$ V. $E_0\ \sqrt{3}=2700$ V.

$i_0=1820$ A.

$p=1410$ KW.

$s_0=.67\%$ $2s_0=1.34\%$

(5.) The 20,000 KW unit in Northwest Station against one 12,000 KW unit in Fisk Street, B.

$x_2 = 5.35$: $2x_1 = 1.356$: $x = .074$: $r = .312$: $z = 1.46$: $M_1 = 50 \times 10^6$: $M_2 = 78 \times 10^6$.

$e_0 = 7330$ V.

$E_0 = .215\ e_0 = 1570$ V.

$E_t = 61\ E_0 = 950$ V. $\quad E_t = \sqrt{3} = 1640$ V.

$i_0 = 2150$ A.

$p = 1690$ KW.

$s_0 = .73\%$. $\quad s_0{}^1 = .59\%$ $\quad s_0 + s_0{}^1 = 1.32\%$

This means, with all the alternators in Fisk B and in Northwest Station out of synchronism with each other, the terminal voltage will be between 1640 and nothing. The interchange current circulating between them will reach maximum amplitudes not exceeding 1820 to 2150 amperes effective. The i^2r loss in the tie cables then would fluctuate between maximum values of 1500 to 2200 KW and nothing, with a probable average of about 900 KW. This agrees with the observation that the wattmeter in the tie lines was very steady showing about 1000 KW, and the voltage was in appreciable.

The conclusion therefore is inevitable, that in this trouble, not only the Northwest Station and the Fisk Street B Station had broken synchronism with each other, but that the individual geneators in the Fisk Street B Station and in the Northwest Station had broken synchronism with each other also, and were drifting past each other out of synchronism.

The important question then is, what caused these alternators and stations to break synchronism.

That the Northwest Station and the Fisk Street B Station drifted out of step with each other was to be expected. As the tie cable which connect the bus bars of these two stations with each other contain appreciable resistance but practically no reactance, and the synchronizing power depends on the reactance, there can be only very little synchronizing power between these stations, that is, in normal synchronous operations of these two stations, it is the individual generators of one station which synchronize with those of the other station, rather than the stations as a whole.

However, the alternators in Fisk Street B, have considerable reactance, and no resistance between each other. The reason for their breaking synchronism with each other must be found in the great drop of voltage resulting from the break of synchronism between Fisk Street B and Northwest Station. The synchronizing

power is proportional to the square of the voltage. With Fisk Street B and Northwest Station out of synchronism with each other but the individual alternators in either station still in synchronism with each other (Case 1), the actual induced voltage in these two stations drops to an average of $E_0 = 1910$ volts per phase. The synchronizing power between two machines, for instance, two of the 12,000 KW alternators of Fisk Street B, reaches a maximum at a phase displacement between two machines of $2\omega = 90$ degrees, and then is:

$$p = \frac{E_0^2}{z} = 2100 \text{ KW per phase,}$$

or a total of 6300 KW, or about half load, and still less in (Case 3), after the 30,000 KW unit had been shut down.

Thus, if the load is suddenly released on these machines, unless the steam supply can be reduced almost instantly to less than half load, these machines will be torn out of synchronism with each other. But by breaking synchronism between machines in the same stations —as the 20,000 and the 30,000 in Northwest Station—the voltage is still further lowered and thereby the synchronizing power reduced, so that, if one machine breaks out of synchronism under these circumstances, all will break. This must have happened on September 18th, 1919. In other words, the break of synchronism between Fisk Street B and Northwest Station lowers the voltage so that there is not sufficient synchronizing power left to keep the individual machines in each station in synchronism with each other.

The important question then arises, what is the minimum machine voltage necessary to assure the individual machines of each station to remain in synchronism with each other irrespective of any sudden release of full load. That is, at what voltage does the synchronizing power of the individual machines become greater than full load. This is gven by:

$$p = \frac{E_0^2}{z} = \frac{1}{3} \text{ rating}$$

as shown in the following table:

TABLE

Lowest Voltage at Which Machines Stay in Synchronism, If Full Load is Thrown Off

Rating of Machine, KW	12,000	14,000	20,000	25,000	30,000	35,000
Voltage per Phase, $E_0 =$	2,630	2,490	2,560	2,740	2,770	2,650
$E_0 \sqrt{3} =$	4,560	4,500	4,430	4,750	4,810	4,600

As seen, as long as the actual induced voltage does not drop below 5000 volts there appears no danger of any of the machines breaking out of synchronism with the other machines in the same station.

The insertion of sufficiently large power limiting reactors between Fisk Street B and Northwest Station would in case of these stations breaking synchronism with each other, maintain sufficient voltage in these stations, so that the machines in each station may be expected to keep in synchronism with each other.

Coming now to the consideration of the relation between Fisk Street B and Quarry Street Station, during the trouble of September 18th, 1919:

(6.) Four 12,000 KW alternators in Fisk Street B, out of synchronism over a power limiting reactor with three 14,000 KW alternators in Quarry Street, the latter in synchronism with each other:

$x_2 = 1.48$: $2x_1 = .438$: $x = 1.75$: $z = 2.19$: $M_1 = 200 \times 10^6$: $M_2 = 200 \times 10^6$.

$e_0 = 6585$ V.

$E_0 = .597\ e_0 = 3930$ V. $\qquad E_0 \sqrt{3} = 6800$ V.

$E_t = \frac{x+z}{2z} E_0 = 3500$ V. $\qquad E_t \sqrt{3} = 6000$ V.

$i_0 = 3600$ A.

$p = 7000$ KW.

$s_0 = s_0^1 = .76\%$ $\qquad s + s_0 = 1.52\%$

As seen, in Quarry Street, the voltage $E_0 \sqrt{3}$ is 6800, thus well above the value required for stability of synchronous operation of the alternators in this station, and no danger existed of their breaking synchronism.

The record shows during the disturbance, while Quarry Street was connected with Fisk Street B out of synchronism, a fairly uniform sustained voltage of 6800, preceded by an initial drop of short duration, below 6000 volts. The latter may be accounted for by the feeding back from Quarry Street over the substations into Fisk B and Northwest Station, which pulled the voltage of Quarry Street down, until the substations cut off, and required some time to recover. The above calculated voltage of 6000 is lower than the observed voltage of 6800. However, Quarry Street was also connected to Fisk Street A, and the latter station was assisting Quarry Street in feeding the current over power limiting reactor B into Fisk B and Northwest Station. We may thus estimate the effect of Fisk A in holding up the voltage.

(7.) Four 12,000 KW alternators in Fisk B, out of synchronism over a power limiting reactor with three 14,000 KW alternators in Quarry Street, the latter in synchronism with each other, and over a second power limiting reactor, with six 12,000 KW alternators in Fisk A.

$x_2 = 1.21$: $2x_1 = .414$: $x = 1.75$: $z = 2.16$: $M_1 = 200 \times 10^6$: $M_2 = 500 \times 10^6$.

$e_0 = 6635$ V.

$E_0 = .64\ e_0 = 4240$ V. $E_0 \sqrt{3} = 7350$ V.

$E_t = 3800$ V. $E_t \sqrt{3} = 6600$ V.

$i_0 = 3900$ A.

$p = 8300$ KW.

$s_0 = .82\%$.

As seen, the calculated value of the terminal voltage of Quarry Street, 6600, agrees as closely with the observed value of 6800 V. as can be expected from such approximated calculations, especially when considering that some synchronous machines had been lost by Quarry Street in the substations, and the load thereby reduced, which would result in an increase of voltage.

As the total impedance of Fisk Street A is about 1.1, and it is connected to Quarry Street by $x = 1.75$, the voltage of Fisk Street A should be higher than Quarry Street in the proportion of 1.1 to 1.1+1.75. This gives for Fisk Street A: $E_t \sqrt{3} = 8100$ V. This well agrees with the average of the voltage record of Fisk Street A during the first seven minutes of the disturbance.

Allowing for the continuously low voltage maintained at Fisk Street B by the out of synchronism condition with Northwest Station, the fluctuating current over the reactor B between Fisk Street B and Quarry Street would vary approximately between 1200 and 2700 amperes. Assuming a temperature rise of these reactors of 30°C. at 600 A., this would give an estimated final temperature rise, at this excessive overload, of 600 to 800°C., so that it should be expected that after seven minutes, when this reactor was disconnected, it would be very hot.

The reactor A, between Quarry Street and Fisk Street A, should carry a current of 500 to 1000 amperes, thus would not become heated.

The conclusion then is: As the calculated numerical values agree with the records of observation as closely as can be expected from

such necessarily approximated calculations, it appears safe to accept the correctness of the explanation that—

Fisk Street A and Quarry Street remained in synchronism with each other, but Fisk Street B and Northwest Station broke out of synchronism with Quarry Street and with each other, and the individual machines of these latter two stations broke out of synchronism with each other, and were unable to pull into step, due to frequency differences greater than permissible by the small synchronising power existing between these machines at the low voltage existing due to the break of synchronism between the stations.

I

In the trouble of *May 19th*, 1919, the following machines were involved:

Fisk A:

Five 12,000 KW=60,000 KW. Load 32,000 KW=53%: 98% Reactance.

$\sqrt{1+.53^2 \times .98^2}=1.13$ $e_0=5900$ V.

$x_2=5.76$: $x_1=.458+.405=.863$: $M=50 \times 10^6$.

Quarry Street:

Four 14,000 KW=56,000 KW. Load 35,000 KW=62.5%: 91% Reactance.

$\sqrt{1+.625^2 \times .91^2}=1.15$ $e_0=6000$ V.

$x_2=4.60$: $x_1=.318+.347=.665$: $M=67 \times 10^6$.

Fisk B:

Three 12,000 KW and one 25,000 KW=61,000 KW. Load 45,000 KW=74%.

98%: >100% Reactance.

$\sqrt{1+.74^2 \times .98^2}=1.24$ $e_0=6450$ V.

$\sqrt{1+.74^2 \times 1.20^2}=1.30$ $e_0=6750$ V.

25,000, estimated: $x_2=4.0$: $x_1=.45$: $M=85 \times 10^6$.

Northwest:

Two 20,000 KW and one 30,000 KW=70,000 KW.

Load 50,000KW=71%.

134%: 125% Reactance.

20,000: $\sqrt{1+.71^2 \times 1.34^2}=1.38$ $e_0=7170$ V.

$x_2=4.95$: $x_1=.325+.168=.493$; $M=78 \times 10^6$.

30,000: $\sqrt{1+.71^2 \times 1.25^2}=1.34$ $e_0=7000$ V.

$x_2=2.99$: $x_1=.214+.168=.382$ $M=87 \times 10^6$.

Busbar reactors: $x=1.75$

Tie cables: $x=.074$; $r=.312$.

Let us assume at first that these stations have dropped out of synchronism with each other, due to the trouble in Fisk A, and are drifting past each other, and calculate by the preceding equations for this condition the voltages and currents in the different stations, and compare them with the recorded values, to see whether this assumption is reasonable. It is then:

(1.) Fisk A out of synchronism with Quarry Street:
$x_2=(1.15+1.15)\div 2=1.15$; $2x_1=.173+.166=.339$:
$x=1.75$ $z=2.09$
$e_0=5950$ V.

$$E_0=\frac{z}{z+x_2}e_0=.645e_0=3840 \text{ V.}$$

$$E_t=\frac{x+z}{2z}E_0=3530 \text{ V.} \qquad E_t\sqrt{3}=6100 \text{ V.}$$

$$i_0=\frac{2E_0}{z}=3670 \text{ A.}$$

(2.) Quarry Street out of synchronism with Fisk B and Fisk A:
$x_2=(1.15+.61)\div 2=.88$; $2x_1=.166+.087=.253$;
$x=.875$; $z=1.128$

$$e_0=1/2\left(5900+\frac{36x6450+25x6750}{36+25}\right)=6240$$

$E_0=.564\ e_0=3520$ V.
$E_t=3120$ V. $E_t\sqrt{3}=5400$ V.
$i_0=6250$ A.

(3.) Fisk B out of synchronism with Northwest Station:
$x_2=(1.30+1.36)\div 2=1.33$; $2x_1=.176+.150=.326$; $x=.074$;
$r=.312$; $z=.507$.
$e_0=6830$ V.
$E_0=.276e_0=1880$ V.
$E_t=1530$ V. $E_t\sqrt{3}=2650$ V.
$i_0=7400$ A.

For the reasons discussed before, it is not probable the individual machines in Fisk B and Northwest Station would stay in synchronism with each other. As the relatively high terminal voltage recorded in the stations shows that the individual machines stayed in synchronism with each other, the case thus may be considered of Fisk B and Northwest Station in synchronism with each other, but out of synchronism with Quarry Street:

(4.) Fisk B and Northwest Station in synchronism with each other, but out of synchronism with Quarry Street:

$x_2=(.683+1.15)\div 2=.92$; $2x_1=.13+.166=.299$; $x=1.75$;
$z=2.05$.
$e_0=6840$ V.
$E_0=.69e_0=4720$ V.
$E_t=4250$ V. $E_t\sqrt{3}=7350$ V.
$i_0=4600$ A.

Herefrom then we get the terminal voltages of the different stations:

	Fisk A:	Quarry:	Fisk B:	Nw. St.:
(1.) All 4 stations out of synchronism:				
Calculated terminal voltage...	6100	5400	2650	2650
Observed records......	7800 to 8200	7900 to 8100 8100 to 8500	8300 to 8400	8400 to 8500
(2.) Fisk B and Nw. Station in synchronism:				
Calculated.....	6100	5400	7350	7350
Observed records......	7800 to 8200	7900 to 8100 8100 to 8500	8300 to 8400	8400 to 8500

As seen, the calculated values are uniformly very much lower than the observed values, and while no very great accuracy can be attributed to such approximated calculations, the difference is too great to be accounted for, and it must therefore be concluded that in this trouble none of the stations had broken synchronism, but all the four stations had kept in synchronism with each other.

Assuming then as reasonable, that in the trouble of May 19th, the four station sections had kept in synchronism, the question remains to account for the great drop and fluctuations of voltage, which were greatest at the source of the trouble, Fisk Street A, and decreased towards the other end of the station chain, whether due to the hunting of the stations against each other, or due to excessive load of lagging current, caused by starting of synchronous machines, or due to some other cause.

Assuming first, that the voltage drop and voltage fluctuations was due to the simultaneous starting of numerous synchronous machines. To estimate the effect thereof, in the following table are given, in columns (1) to (3), the recorded voltages of the four stations:

The initial minimum and the final minimum towards the end of the disturbance, and the average fluctuation. In the second line is given the average phase voltage, that is terminal voltage divided by $\sqrt{3}$. Column (4) then gives the average drop of voltage; column (5) the total station impedance, true reactance as well as effective reactance of armature reaction. As we have to do here with a sustained voltage drop, the armature reaction comes into play. Column (6) then gives the average value of the lagging current in each station, which would account for this voltage drop. As seen these are fairly moderate currents. This would be the lagging current drawn from the stations in starting the synchronous machines in the substations. As (practically) all the synchronous machines in Fisk A had dropped out, but only a few on the other stations, most of this lagging load would be on Fisk Street A. Assuming then that half this current had been consumed directly from the stations, the other half transferred over the power limiting reactors to Fisk Street A. Column (7) would then give the average lagging current consumed in each station, and Column (8) the current transferred over the power limiting reactors between the stations, to the next station, and Column (9) gives the load on each station, in KVA. As Fisk Street A lost a load of about 32,000 KW in synchronous machines, a load of 12,000 KVA starting current of these synchronous machines, during eighteen minutes, does not appear excessive, and the lesser values in the other stations also appear of reasonable magnitude. The voltage across the power limiting reactors, in Column (10). and the phase angles between the stations, in Column (11), are very moderate: only a little over 5 degrees maximum phase displacement between Fisk A and Quarry Street. That is, the strain is very moderate, and very far from the limits of synchronizing power.

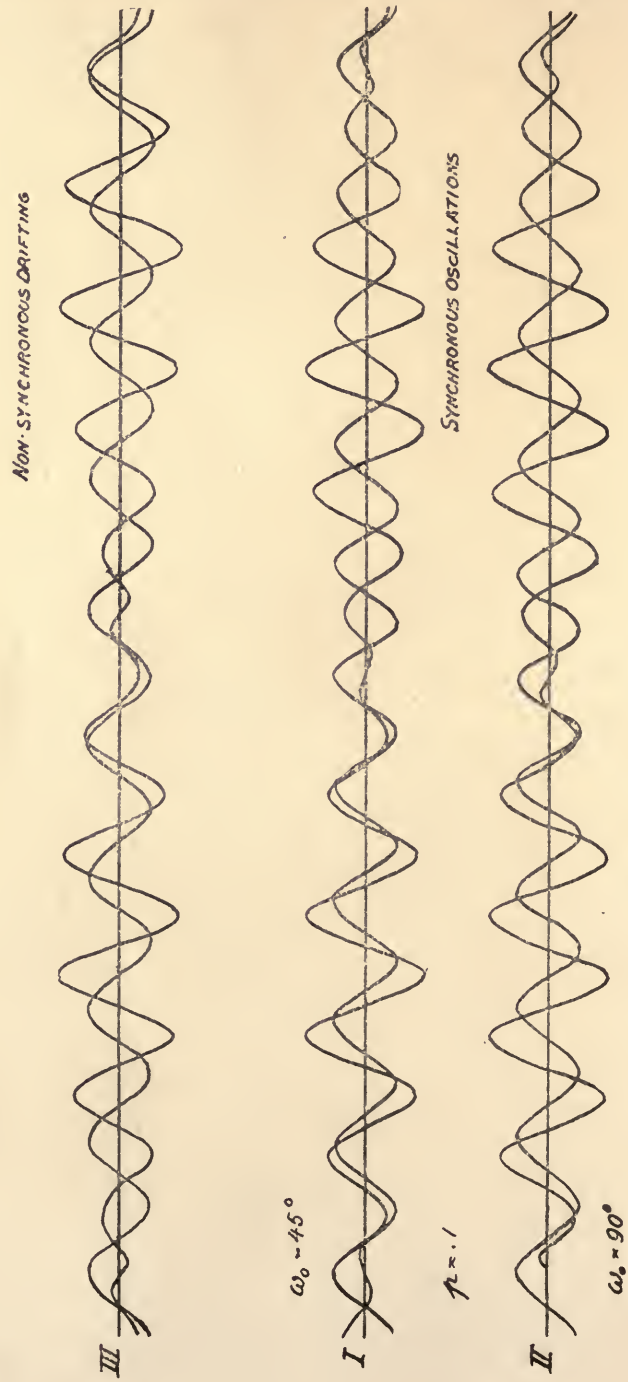
Assuming now the second explanation, hunting of the stations. Column (12) then gives the estimated values of the quadrature voltage resulting from the swing of the machines, which would be required to bring about the observed voltage drop: $\sqrt{5200^2 - e^2}$ where 5200 is the normal phase voltage of the stations, and e the observed phase voltage. The total phase angle of swing then is given as $2\omega^1$ in Column (13). As the limits of synchronizing power are at 90 degrees phase angle, and the maximum estimated phase angle of swing is 30 degrees, it would appear—if this explanation is correct—that the stations were still far from the limits of their synchronizing power, that is, quite stable. Column (14) then gives the true self inductive reactance of the stations, and Column (15)

gives two values of the surging current in the stations resulting from the oscillations of the machines. The lower value is calculated by the total station impedance, Column (5), the higher value by the self inductive impedance Column (14). The true value would be intermediate, probably nearer the larger figure, as the period of oscillation is too fast to develop more than a small part of the effective reactance of armature reaction.

Either of the two assumptions gives values of magnitude which are reasonable and in accordance with the observed data, so that in the absence of any data on the phase angle between the stations, it is not possible to decide which is the correct explanation. It must be recognized, however, that neither of the two explanations is entirely satisfactory, as either fails to account sufficiently for the excessive heating of the reactor B between Quarry Street and Fisk Street B. This makes it the more desirable, in view of the importance of fully understanding the phenomenon, to install phase angle indicators, that is suitable synchronoscopes, in all the stations, and carefully observe them in case of any trouble in the system.

TABLE

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	Voltage Record														
Station	Min.	Max.	Fluct.	Average Voltage Drop per Phase	Station Impedance Ohms.	Average Current i:	Current Consumed i_1:	Current Transferred i_2:	Average Load KVA. $ei_1 \sqrt{3}$	React. Volts xi_2	Phase Angle 2ω	$\sqrt{5200^2-e^2}$	Phase Angle $2\omega'$:	x_1:	i':
Fisk A.........	7800	8200	±700	580±400	1.32	440	870		12000			2390	30°	.173	1800 14000
	4620							430		750	5.3°				
Quarry.........	8000	8300	±300	490±170	1.31	380	190		2650			2220	27°	.166	1700 13400
	4710							240		420	3.0°				
Fisk B.........	8300	8400	±400	380±230	1.47	260	130		1800			1950	24°	.176	1300 10500
	4820							110		35	—				
Nw. St.........	8400	8500	±200	320±120	1.50	220	110		1500			1800	21°	.150	1200 12000
	4880														


NON-SYNCHRONOUS DRIFTING
SYNCHRONOUS OSCILLATIONS
$\omega_0 = 45°$
$p = .1$
$\omega_0 = 90°$
III
I
II